CRRA-utility

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The Constant Relative Risk Aversion utility function is

$$u(c) = \begin{cases} \frac{1}{1-\theta} c^{1-\theta} & \text{if } \theta > 0, \theta \neq 1 \\ \ln c & \text{if } \theta = 1 \end{cases}$$

Taking derivatives we find

$$u'(c) = c^{-\theta}$$

Hence,

$$\frac{u'(c_1)}{u'(c_2)} = \frac{c_1^{-\theta}}{c_2^{-\theta}} = \left(\frac{c_2}{c_1}\right)^{\theta}$$

or, solving for c_2/c_1.:

$$\frac{c_2}{c_1} = \left(\frac{u'(c_1)}{u'(c_2)}\right)^{1/\theta}$$

Here, $\sigma = 1/\theta$ is the elasticity of the ratio between the consumed quantities of the two goods with respect to the marginal rate of substitution. By definition σ is then the elasticity of substitution, which is constant for the CRRA utility function. σ is a measure of the strength of the substitution effect that a change in relative prices induces. In the context of the Ramsey model a low σ means a strong preference for avoiding inequality between generations in excess of what follows from the discounting in the utility function.

The first order condition (Euler equation) in the Ramsey model was:

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta[1 + f'(k_{t+1})]$$

With CRRA utility this becomes

$$\left(\frac{c_{t+1}}{c_t}\right)^{\theta} = \beta[1 + f'(k_{t+1})]$$

which means that the consumption growth rate is

$$\frac{c_{t+1}}{c_t} = \left[\beta(1 + f'(k_{t+1}))\right]^{1/\theta} = \left(\frac{1 + f'(k_{t+1})}{1 + \rho}\right)^{\sigma}$$

A high σ means the difference between the marginal productivity of capital and the subjective discount rate has a strong effect on the consumption growth rate.