ECON4310 Exercise 3

Due 13/9 2010

Optimal growth with equal weight for all generations

In lecture 5 the social planner placed higher weight on the utility of large generations. Assume that the planner's utility function is instead

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t A_t) \tag{1}$$

This is to be maximized subject to

$$c_t = k_t + f(k_t) - (1+n)(1+g)k_{t+1}, \tag{2}$$

and

$$k_0 = \bar{k_0}, k_t \geq 0, c_t \geq 0$$

The period utility function is CRRA:

$$u(x) = (1/(1-\theta))x^{1-\theta}, \quad \sigma = 1/\theta > 0 \tag{3}$$

You can assume that $\sigma < 1$.

1. Write down the value function and the Bellman equation for the problem.

2. Derive the first-order condition for optimum and show that it can be written
$$\frac{c_{t+1}}{c_t} = \left[\frac{1+f'(k_{t+1})}{(1+\rho)(1+n)}\right]^{\sigma} \frac{1}{1+g}, \quad t = 1, 2, \ldots$$

3. What determines the capital intensity k_* along a balanced growth path? Compare k_* to the golden rule level of k and the level we found in lecture 5. (Loglinearize if you like). What is the intuitive reason for the difference between the three cases?

4. Along the balanced growth path, what are the growth rates of a) consumption per efficiency unit of labor?, b) consumption per capita?, c) total consumption ?

5. Draw a phase diagram for the model. Illustrate what the optimal path will look like when $k_0 > k_*$. Explain briefly how the starting point is pinned down.

6. Draw a new phase diagram with $k_0 = k_*$. Suppose the productivity growth rate increases. How does the two equilibrium curves shift? Where is the new stationary point located relative to the old? Describe the path on which the planner will take the economy to the new steady state. Where does it start from?