ECON4310 Exercise 1

Due 30/8 2010

This exercise uses the static competitive equilibrium model from Lecture 1 and Williamson's notes chapter 1.1. You are asked to look at the special case where the utility function of the representative consumer is

$$U = \ln c + \mu \ln \ell \tag{1}$$

and the production function of the representative firm is

$$y = zk^{1-\alpha}n^{\alpha} \tag{2}$$

Here $\mu > 0$, and $0 < \alpha < 1$ are constant parameters. For simplicity, the number of consumers and producers is set equal to one.

1. We first look at consumer behavior. the budget equation of the consumer is

   $$c = w(1-\ell) + y_0 \tag{3}$$

   Here, $y_0 = r\bar{k}$ is the revenue from the consumer's initial endowment of capital, and $1-\ell$ is labor supply.

   Derive the first-order condition for maximum utility. Use it to answer how the ratio between consumption and leisure (c/ℓ) is affected

   (a) if the wage rate increases by ten per cent?

   (b) if the unearned income y_0 increases by ten per cent?

   Solve for consumption demand and labor supply as functions of w and y_0.

2. Write down the marginal productivity conditions that characterize firm behavior in equilibrium. Show that they imply that the share of labor income in output, wn/y is equal to α.

3. In equilibrium the marginal rate of substitution between consumption and leisure has to be equal to the marginal rate of transformation. Show that this condition is the same as

   $$\frac{\mu c}{\ell} = \alpha z \left(\frac{k}{1-\ell}\right)^{1-\alpha} \tag{4}$$

   The equilibrium must also be on the production function (2). Use the two equations together with the market-clearing conditions $c = y$ and $k = \bar{k}$ to solve for c and ℓ. Hint: Start by using (2) to eliminate c from (4).

4. In question 4 you will discover that the equilibrium value of ℓ is independent of z and k. Give an intuitive reason for this result. (Hint: Draw on the answer to question 1). Does the result fit the historical facts?

5. We now include government consumption, g, in the model. Government consumption enters the utility function additively ($U = u(c, \ell) + v(g)$). Suppose that the government sets g equal to a share γ of output. It is financed with a lump-sum tax on the individuals, $t = g = \gamma y$. This means that now $y_0 = r\bar{k} - t$. Explain why in equilibrium the marginal rate of substitution will still be equal to the marginal rate of transformation as in (4). However, instead of $c = y$, we now have $c = (1 - \gamma)y$. Show that this changes the equilibrium amount of leisure to

$$\ell = \frac{\mu(1-\gamma)}{\alpha + \mu(1-\gamma)} \tag{5}$$

Will a larger government sector lead to more or less supply of labor? Why?