Macroeconomic Theory

Econ 4310 Lecture 2

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Solow's growth model

- Source: Romer Ch. 1
- Discrete versus continuous time
- Read Ch. 1

Solow's growth model

$$Y_t = F(K_t, A_t L_t) \qquad (1)$$

$$I_t = sY_t \qquad (2)$$

$$K_{t+1} = K_t + I_t \qquad (3)$$

$$L_t = L_0(1+n)^t \qquad (4)$$

$$A_t = A_0(1+g)^t \qquad (5)$$

Y_t = output
K_t = capital,
L_t = labor
I_t = investment
A_t = labor productivity

F = production function
F: constant returns to scale,
$F_1, F_2 > 0, F_{11}, F_{22} < 0$
K_0, L_0 given

Stocks and flows

- Flows are measured per unit of time
  - I_t no of machines per year
- Stocks are measured at a point in time
  - K_t no of machines available at beginning of period t
- Some flows are add to stocks over time
  - Accumulation equations: $K_{t+1} = K_t + I_t$

Condensing the model (Removing the trend)

$A_t L_t$ = Labor input in efficiency units.

$$A_t L_t = A_0 L_0 [(1+g)(1+n)]^t = A_0 L_0 (1+\gamma)^t$$

γ = "natural" growth rate, $\gamma = n + g + ng$ Define new variables:

$k = K/AL$ = capital intensity, $y = Y/AL$ = output per efficiency unit of labor

$$y = F(K_t, A_t L_t)/A_t L_t = F\left(\frac{K_t}{A_t L_t}, \frac{A_t L_t}{A_t L_t}\right) = F(k_t, 1)$$

Define $f(k) = F(k, 1)$. Then

$$y_t = f(k_t) \tag{6}$$

Condensing the model

$$K_{t+1} - K_t = I_t = sY_t \tag{7}$$

Divide through (7) by $A_t L_t$:

$$\frac{K_{t+1}}{A_t L_t} - \frac{K_t}{A_t L_t} = s\frac{Y_t}{A_t L_t}$$

$$k_{t+1}(A_{t+1}L_{t+1}/A_t L_t) - k_t = sy_t$$

or

$$k_{t+1}(1+\gamma) - k_t = sy_t \tag{8}$$

γk_{t+1} = investment needed for capital stock to keep pace with natural growth rate

Model in intensive form

$$k_{t+1}(1+\gamma) - k_t = sy_t$$
$$y_t = f(k_t)$$

or simply

$$k_{t+1}(1+\gamma) - k_t = sf(k_t) \tag{9}$$

One difference equation in one unknown time series, k_t.
Initial k (k_0) given

Assumed properties of f (Inada conditions):

$$f'(k) > 0, \;\; f''(k) < 0$$
$$f(0) = 0, \;\; f'(0) = \infty, \;\; f'(\infty) = 0$$

Wages and rental price of capital:

$$r_t = F_1(K_t, A_t L_t) = f'(k_t)$$

$$w_t = A_t F_2(K_t, A_t L_t) = A_t[f(k_t) - k_t f'(k_t)]$$

(Use Euler's theorem and that when $F(X_1, X_2)$ is homogeneous of degree 1, then $F_i'(X_1, X_2)$ is homogenous of degree 0).

The steady state ("Balanced growth path")

$$k_{t+1}(1+\gamma) - k_t = sf(k_t)$$

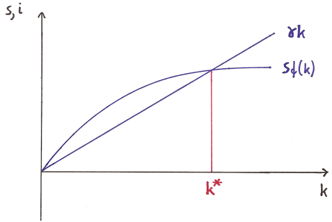
Steady state k^* defined by $k_{t+1} = k_t = k^*$ or

$$k^*(1+\gamma) - k^* = sf(k^*)$$

$$sf(k^*) = \gamma k^* \qquad (10)$$

γk^* = investment needed to keep up with growth in AL.

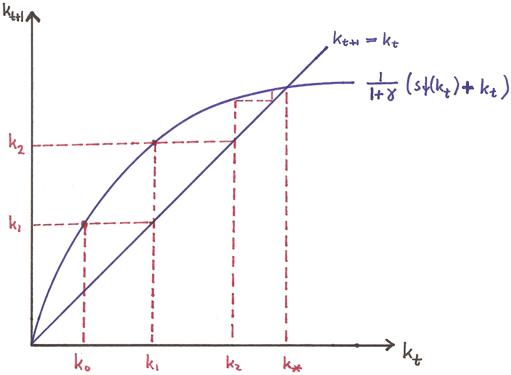
Two steady states, one with $k^* = 0$, one with $k^* > 0$



More about the steady state

- Growth rate of output per capita is equal to productivity growth rate g
- Capital intensity, k^* depends positively on s, negatively on n and g.
- Level of output depends positively on s, negatively on n
- Real interest rate, $r^* = f'(k^*)$ depends negatively on s, positively on n and g
- Real wage, $w^* = A[f(k^*) - k^* f'(k^*)]$, grows with rate of productivity growth
- The share of wage income in total output is constant.
- Level of real wage depends positively on s, negatively on n and g

Transitional dynamics



Stationarity and stability

- A stationary state is in a dynamic model is a state where all the variables in the model stay constant
- A stationary state is a state that reproduces itself over time
- In $k_{t+1} = [sf(k_t) + k_t]/(1+\gamma)$ $k_t = k^*$ makes $k_{t+1} = k^*$
- A stationary state can be either stable or unstable

Stability

- A stationary point is globally stable if, for any given starting point, the economy moves towards that stationary point as time goes to infinity
- A stationary point is locally stable if for any starting point *in a region around the stationary point*, the economy moves towards that stationary point as time goes to infinity.
- In a single equation model local stability requires that the feed-back from the variable to itself is less than one-to-one.

$$\frac{dk_{t+1}}{dk_t} = \frac{sf'(k_t)}{1+\gamma} < 1$$

The Golden Rule of Accumulation

Consumption per efficiency unit of labor in steady state is:

$$c = f(k) - \gamma k \tag{11}$$

First order condition for maximum is $f'(k) - \gamma = 0$. Golden rule level of k, k^{**} is determined by

$$f'(k^{**}) = \gamma \tag{12}$$

$$r^{**} = \gamma$$

Interest rate equal to natural growth rate
Savings rate required to reach k^{**}:

$$s^{**} = \gamma k^{**}/f(k^{**}) = r^{**}k^{**}/f(k^{**})$$

Along the Golden rule path the savings rate equals the income share of capital.
If s is increased beyond s^{**}, consumption is reduced both now and in all future!

Some questions

1. Is it conceivable that rational agents will save too much for society's long-run good?
2. Should society aim at the golden rule level of capital in the long run?
3. How much private saving should we expect in a market equilibrium?

Discrete versus continuous time

$$k_{t+1} - k_t = sf(k_t) - \gamma k_{t+1}$$

$$sf(k^*) = \gamma k^*$$

$$\gamma = n + g + ng$$

$$\dot{k}(t) = sf(k(t)) - \gamma k(t)$$

$$sf(k^*) = \gamma k^*$$

$$\gamma = n + g$$