Intertemporal macroeconomics

Econ 4310 Lecture 5 Part 1

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Ramsey-model with trend growth

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t A_t) L_t \text{ given} \tag{1}$$

$$c_t = k_t + f(k_t) - (1+n)(1+g)k_{t+1}, \tag{2}$$

$$A_t = A_0(1+g)^t, \qquad L_t = L_0(1+n)^t$$

$$k_0 = \bar{k_0}, k_t \geq 0, c_t \geq 0$$

Maximize with respect to c_t and k_{t+1} for $t = 0, 1, 2, \ldots$

Using Bellman's method

Value function:

$$V(k_t, A_t, L_t) = \max \sum_{j=0}^{\infty} \beta^j u(c_{t+j} A_{t+j}) L_{t+j} \tag{3}$$

Bellman equation:

$$V(k_t, A_t, L_t) = \max_{c_t, k_{t+1}} \left[u(c_t A_t) L_t + \beta V(k_{t+1}, A_{t+1}, L_{t+1}) \right] \tag{4}$$

- Maximization is subject to constraints above
- Imagine that we substitute for c_t from (2) in (4)

Deriving first order conditions

$$\max_{k_{t+1}} [u([k_t + f(k_t) - (1+\gamma)k_{t+1}]A_t)L_t + \beta V(k_{t+1}, A_{t+1}, L_{t+1})]$$

Differentiate with respect to k_{t+1}:

$$u'(c_t A_t)[-(1+\gamma)]A_t L_t + \beta V_1'(k_{t+1}, A_{t+1}, L_{t+1}) = 0$$

Because optimization takes place also in the future:

$$V_1'(k_{t+1}, A_{t+1}, L_{t+1}) = u'(c_{t+1}A_{t+1})A_{t+1}L_{t+1}(1 + f'(k_{t+1}))$$

Combine the two above!

$$-u'(c_tA_t)\overbrace{(1+\gamma)A_tL_t}^{A_{t+1}L_{t+1}}+\beta u'(c_{t+1}A_{t+1})A_{t+1}L_{t+1}(1+f'(k_{t+1}))=0$$

which simplifies to

$$u'(c_tA_t)=\beta u'(c_{t+1}A_{t+1})(1+f'(k_{t+1})) \tag{5}$$

Warnings!

- There may be no maximum!
- Discounted utility may be infinite! Low discount rate, high productivity growth
- Balanced growth paths are possible only if $u(c)$ is CRRA

Assume CRRA utility and $\rho > (1 - \theta)g + n$.

CES-preferences

$$u(x) = (1/(1-\theta))x^{1-\theta}, \sigma = 1/\theta > 0$$

First order condition:

$$(c_t A_t)^{-\theta} = (c_{t+1} A_{t+1})^{-\theta} \beta(1 + f'(k_{t+1}))$$

$$\frac{(c_t A_t)^{-\theta}}{(c_{t+1} A_{t+1})^{-\theta}} = \beta(1 + f'(k_{t+1})) \tag{6}$$

$$\frac{c_{t+1} A_{t+1}}{c_t A_t} = [\beta(1 + f'(k_{t+1}))]^{\sigma}$$

Consumption growth rates

Per capita:

$$\frac{c_{t+1}A_{t+1}}{c_t A_t} = [\beta(1 + f'(k_{t+1}))]^{\sigma}$$

Per efficiency unit of labor:

$$\frac{c_{t+1}}{c_t} = [\beta(1 + f'(k_{t+1}))]^{\sigma}/(1 + g)$$

Conditions for balanced growth

$$c_{t+1} = c_t \Longrightarrow \quad \left(\frac{1+f'(k_*)}{1+\rho}\right)^{\sigma}\frac{1}{1+g} = 1 \tag{7}$$

$$k_{t+1} = k_t \Longrightarrow \quad c^* = f(k^*) - (n+g+ng)k_* \tag{8}$$

Loglinearizing steady state condition (7)

$$(1+g)^{-1}(1+\rho)^{-\sigma}[(1+f'(k_*))]^{\sigma} = 1$$

$$-\ln(1+g) - \sigma\ln(1+\rho) + \sigma\ln(1+f'(k_*)) = \ln 1 = 0$$

$$-g - \sigma\rho + \sigma f'(k_*) \approx 0$$

$$f'(k_*) \approx \rho + \frac{1}{\sigma}g \tag{9}$$

Short period, g, ρ and $f'(k)$ small numbers, $ln(1+x) \approx x$

σ	ρ	g	$f'(k_*)$
0.5	0.02	0.03	0.08
0.5	0.02	0.04	0.10
1.0	0.02	0.03	0.05
1.0	0.02	0.02	0.04

Observations on the steady state

$$f'(k_*) \approx \rho + \frac{1}{\sigma} g$$

- k_* is independent of n
- k_* depends negatively on ρ
- k_* depends negatively on g and more so the lower is σ
- k_* depends positively on σ when $g > 0$

High g implies high real interest rate.

Comparison of steady states

$$\text{Golden rule } f'(k_{**}) \approx n + g$$

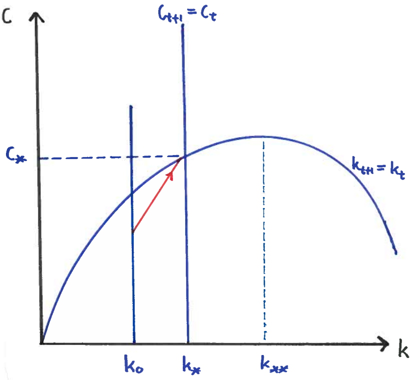
$$\text{Ramsey } f'(k_*) \approx \rho + \frac{1}{\sigma} g$$

By assumption

$$\rho > \left(1 - \frac{1}{\sigma}\right) g + n$$

By implication

$$\rho + \frac{1}{\sigma} g > n + g \qquad \text{and} \qquad k_* < k_{**}$$



$$c_= f(k) - \gamma$$