ECON4310 Exercise 3

Due 19/9 2011

1. **Ramsey model with log utility**

   Suppose a social planner wants to maximize

$$\sum_{t=0}^{\infty} \beta^t \ln c_t \tag{1}$$

   given

$$c_t = k_t^{\alpha} + k_t - k_{t+1}, \qquad t = 0, 1, 2, \dots \tag{2}$$

   and given $k_0 = \bar{k}_0$ and $k_t \geq 0$ for $t = 1, 2, \dots$ The natural growth rate is zero, while $0 < \beta < 1$ and $0 < \alpha < 1$.

   (a) Derive the first-order conditions for optimum.

   (b) What determines whether consumption will be growing or declining over time?

   (c) How is the steady state capital intensity in this economy determined? Explain in words why capital accumulation stops before the marginal productivity of capital is zero.

   (d) What is the savings rate in the steady state?

   (e) Suppose $\alpha = 0.3$ and $\beta = 0.96$. What are then the steady state levels of k and y? How much would they differ if the social planner were more patient and had $\beta = 0.98$?

2. **Optimal growth with equal weight for all generations**

   In the lectures the social planner placed higher weight on the utility of large generations. Assume that the planner's utility function is instead

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t A_t) \tag{3}$$

   This is to be maximized subject to

$$c_t = k_t + f(k_t) - (1+n)(1+g)k_{t+1}, \tag{4}$$

   and

$$k_0 = \bar{k_0}, k_t \geq 0, c_t \geq 0$$

The period utility function is CRRA:

$$u(x) = (1/(1-\theta))x^{1-\theta}, \qquad \sigma = 1/\theta > 0 \tag{5}$$

You can assume that $\sigma < 1$.

(a) Derive the first-order condition for optimum and show that it can be written

$$\frac{c_{t+1}}{c_t} = \left[\frac{1+f'(k_{t+1})}{(1+\rho)(1+n)}\right]^{\sigma} \frac{1}{1+g}, \qquad t = 1, 2, \ldots$$

(b) What determines the capital intensity k_* along a balanced growth path? Compare k_* to the golden rule level of k and the level we found in the lectures when each generation's utility was weighted by its size. (Loglinearize if you like). What is the intuitive reason for the difference between the three cases?

(c) Along the balanced growth path, what are the growth rates of i) consumption per efficiency unit of labor?, ii) consumption per capita?, iii) total consumption ?

(d) Draw a phase diagram for the model. Illustrate what the optimal path will look like when $k_0 > k_*$. Explain briefly how the starting point is pinned down.

(e) Draw a new phase diagram with $k_0 = k_*$. Suppose the productivity growth rate increases. How does the two equilibrium curves shift? Where is the new stationary point located relative to the old? Describe the path on which the planner will take the economy to the new steady state. Where does it start from?