

ECON4310 Exercise 1

Due 29/8 2011

This exercise uses the static competitive equilibrium model from Lecture 1 and Williamson's notes chapter 1.1. You are asked to look at the special case where the utility function of the representative consumer is

$$U = \ln c + \mu \ln \ell \quad (1)$$

and the production function of the representative firm is

$$y = zk^{1-\alpha}n^\alpha \quad (2)$$

Here $\mu > 0$, and $0 < \alpha < 1$ are constant parameters. For simplicity, the number of consumers and producers is set equal to one.

1. We first look at consumer behavior. the budget equation of the consumer is

$$c = w(1 - \ell) + y_0 \quad (3)$$

Here, $y_0 = r\bar{k}$ is the revenue from the consumer's initial endowment of capital, and $1 - \ell$ is labor supply.

Derive the first-order condition for maximum utility. Use it to answer how the ratio between consumption and leisure (c/ℓ) is affected

- (a) if the wage rate increases by ten per cent?
- (b) if the unearned income y_0 increases by ten per cent?

Solve for consumption demand and labor supply as functions of w and y_0 .

2. Write down the marginal productivity conditions that characterize firm behavior in equilibrium. Show that they imply that the share of labor income in output, wn/y is equal to α .
3. In equilibrium the marginal rate of substitution between consumption and leisure has to be equal to the marginal rate of transformation. Show that this condition is the same as

$$\frac{\mu c}{\ell} = \alpha z \left(\frac{k}{1 - \ell} \right)^{1-\alpha} \quad (4)$$

The equilibrium must also be on the production function (2). Use the two equations together with the market-clearing conditions $c = y$ and $k = \bar{k}$ to solve for c and ℓ . Hint: Start by using (2) to eliminate c from (4).

4. In question 4 you will discover that the equilibrium value of ℓ is independent of z and k . Give an intuitive reason for this result. (Hint: Draw on the answer to question 1). Does the result fit the historical facts?
5. We now include government consumption, g , in the model. Government consumption enters the utility function additively ($U = u(c, \ell) + v(g)$). Suppose that the government sets g equal to a share γ of output. It is financed with a lump-sum tax on the individuals, $t = g = \gamma y$. This means that now $y_0 = r\bar{k} - t$. Explain why in equilibrium the marginal rate of substitution will still be equal to the marginal rate of transformation as in (4). However, instead of $c = y$, we now have $c = (1 - \gamma)y$. Show that this changes the equilibrium amount of leisure to

$$\ell = \frac{\mu(1 - \gamma)}{\alpha + \mu(1 - \gamma)} \quad (5)$$

Will a larger government sector lead to more or less supply of labor? Why?