14 Consumption and saving

- Review the life-cycle/permanent-income hypothesis of consumption

- Derive Hall's (1978) random walk result

- Discuss failures of the random walk result

- Where do we stand now?

Departure from Keynes

- Optimization is foreward-looking

- Saving today is future consumption; Saving and borowing is used to smooth the path of consumption

- Current consumption does not follow current income; departure from the Keynesian model where

$$C_t = C(Y_t), \text{ estimated as } C_t = a + bY_t + u_t$$

Lifetime utility

Finite horizon (Romer)

$$\sum_{t=1}^{T} u(C_t)$$

Infinite horizon (Williamson)

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

Period utility $u(.)$ is increasing and strictly concave: $u' > 0, u'' < 0$, C_t is consumption in period t.

β is the discount factor, where $\beta = 1/(1+\rho)$ and ρ is the discount rate/time preference rate. A positive ρ reflects impatience or time preference.

Intertemporal budget constraint

$$A_{t+1} = (1 + r)\,(A_t + Y_t - C_t)$$

where r is constant, A are assets. Lifetime budget constraint under:

- Finite horizon

$$\sum_{t=1}^{T} C_t \leq A_0 + \sum_{t=1}^{T} Y_t, \quad A_T = 0$$

- Infinite horizon

$$\begin{aligned}
\lim_{t\to\infty} \frac{A_t}{(1+r)^t} &= 0 \quad \text{(No-Ponzi-scheme)} \\
\sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t} &\leq A_0 + \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t}
\end{aligned}$$

Optimization

The optimization problem

$$\max_{\{C_t\}} \mathcal{L} = \sum_{t=0}^{T-1} \beta^t u(C_t) - \lambda \left[A_0 + \sum_{t=0}^{T-1} \frac{Y_t}{(1+r)^t} - \sum_{t=0}^{T-1} \frac{C_t}{(1+r)^t} \right]$$

First order conditions for C_t and C_{t+1}:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial C_t} &= u'(C_t) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial C_{t+1}} &= \beta (1+r) u'(C_{t+1}) - \lambda = 0 \end{aligned}$$

Euler equation

The intertemporal Euler equations are given by

$$\begin{aligned} \beta(1+r)\,u'(C_{t+1}) &= u'(C_t)\,,\ t = 0, ..., T-1 \\ \frac{u'(C_t)}{\beta u'(C_{t+1})} &= 1+r \\ u'(C_{t+1}) &= \left(\frac{1+\rho}{1+r}\right) u'(C_t) \end{aligned}$$

Consumption grows over time if $r > \rho$. If $r = \rho$, then

$$\begin{aligned} u'(C_t) &= u'(C_{t+1}) \\ C_t &= C_{t+1} \end{aligned}$$

Permanent income

Optimal consumption in every period:

$$C^* = \frac{1}{T}\left(A_0 + \sum_{t=0}^{T-1} \frac{Y_t}{(1+r)^t}\right) = Y^P$$

If $r = \rho = 0$ this becomes:

$$C^* = \frac{1}{T}\left(A_0 + \sum_{t=0}^{T-1} Y_t\right)$$

Saving will then be the difference between current income and permanent income (transitory income):

$$\begin{aligned} S_t &= Y_t - C^* \\ &= Y_t - Y^P \end{aligned}$$

Transitory and permanent changes in income

An *unanticipated* and *transitory* change in income

$$\Delta C_t = \frac{1}{T}\Delta Y_t$$

where Δ denotes an absolute change.

An *unanticipated* and *permanent* change in income

$$\Delta C_t = \frac{1}{T}\sum_{s=t}^{T-1}\Delta Y_s$$

If income change is *anticipated*, consumption does not change at all.

Interpreting the estimated coefficient $\hat{b}$

Friedman distinguishes permanent income, Y^P, from transitory income, Y^T. Current income is $Y = Y^P + Y^T$ and optimal consumption is $C = Y^P$. Then the regression coefficient (from page 2) can be interpreted as follows:

$$\hat{b} = \frac{cov\left(Y, C\right)}{var\left(Y\right)} = \frac{cov\left(Y^P + Y^T, Y^P\right)}{var\left(Y^P + Y^T\right)} = \frac{var\left(Y^P\right)}{var\left(Y^P\right) + var\left(Y^T\right)}$$

The intercept $\hat{a}$ is then

$$\hat{a} = \bar{C} - \hat{b}\bar{Y} = \bar{Y}^P + \hat{b}\left(\bar{Y}^P + \bar{Y}^T\right) = \left(1 - \hat{b}\right) Y^P$$

where bars above variables denote mean values.

Life-cycle model under uncertainty

In each period consumption is chosen so as to maximize

$$E_t \left[\sum_{t=s}^{T-t} \beta^s u\left(C_{t+s}\right) \right]$$

given

$$A_{t+1} = (1+r)\left(A_t + \tilde{Y}_t - C_t\right)$$

where $\tilde{Y}$ is stochastic income (the source of uncertainty). This yields the stochastic Euler equation

$$E_t \left[u'\left(C_{t+1}\right) \right] = \beta (1+r)\, u'\left(C_t\right)$$

Hall's (1978) random walk result

Took the permanent income hypothesis to its extreme by assuming *rational expectations*. Consumers use all available infoemation up to the current time t and incorporate it into their lifetime consumption plan. Formally,

$$X_t^e = E_{t-1}\left[X_t|\Omega_{t-1}\right]$$

where superscript e denotes expectation and Ω_{t-1} is the information set at time $t-1$. This implies that changes in X_t are unpredictable:

$$X_t = X_t^e + \epsilon_t$$

where ϵ_t is an expectations error. A special case is perfect foresight, $X_t^e = X_t$, which says that households expect the outturn that actually holds.

Hall's (1978) random walk result

Assuming quadratic utility (and constant $r = \rho = 0$)

$$E_t \left[\sum_{t=0}^{T-1} C_t - \frac{a}{2} C_t^2 \right], \ a > 0$$

The stochastic Euler equation can the be reduced to

$$\begin{aligned} E_t \left(1 - aC_{t+1}\right) &= 1 - aC_t \\ E_t C_{t+1} &= C_t \\ C_{t+1} &= C_t + \varepsilon_{t+1}, \ E_t \varepsilon_{t+1} = 0 \end{aligned}$$

Random walk hypothesis: Only current consumption is required to predict future consumption.

Failure of the random walk hypothesis

- Tests of "only current consumption is required to predict future consumption" is rejected.

More general shortcomings of the model:

- *Excess sensitivity of consumption*: even anticipated changes in income lead to predictable changes in consumption.

- *Excess smoothness of consumption*: unanticipated permanent changes in income seem to lead to too small responses in consumption.

- A large fraction of households consume all of their income in each period.

Precautionary saving

Hall's results based on quadratic utility gives *certainty equivalence*: consumption depends only expected future income and not uncertainty about that income.

For other functional forms of u in which

$$u''' > 0$$

(the marginal utility is strictly convex), optimal C_t also depends on the variability of the income stream. It follows that

$$E_t\left[u'(C_{t+1})\right] > u'\left[E_t(C_{t+1})\right]$$

Greater uncertainty about future income leads to reduced consumption today and more precautionary saving.

Liquidity constraints

Not all individuals are able to borrow as much as they would like, and at the same interest rate as they can save. Credit markets are imperfect: the borrowing rate exceeds the savings rate, there may be quantity constraints on borrowing, or collateral constraints (mortgages).

Impose a constraint in the model, for instance

$$A_t \geq 0, \ \forall t$$

When the constraint binds, consumption will be equal to current income, and any increases in income will be fully consumed.

Buffer stock saving

In the US: Most households have little wealth. Consumption approximately tracks income, but small amounts of saving are held in the event of income falls or emergency spending. Most households exhibit *buffer-stock* saving behavior.

Assume impatient consumers ($\rho > r$)

Deaton (1991): general utility function and income process, but a liquidity constraint.

Carroll (1997): CRRA utility function and an income process with the possibility of zero income, $\Pr[Y_t = 0] > 0$.

Both models give buffer stock saving behavior.

Where are we now?

CRRA utility function (with adjustment for family composition, which explains hump-shape over the life-cycle)

Stochastic income process (log income follows a random walk with drift)

$$Y_t^P = g_t Y_{t-1}^P \tilde{\eta}_t$$

Precautionary saving through the possibility of zero income, $\Pr[Y_t = 0] > 0$

Bequest motives

This yields "non-analytical" models. Full models can be calibrated (numerical dynamic stochastic programming methods), else just use first order conditions (Euler equations).