Exercise: Demand for and pricing of risky assets

We are looking at a consumer who lives for two periods. He starts life with no assets and leaves no bequests. His consumption in the two periods is C_0 and C_1. His earned income is Y_0 in the first period and Y_1 in the second period. In the first period he has the opportunity to invest in two assets which yield gross returns in the second period $1 + r_a$ and $1 + r_b$. The amount he invests in the first asset is A_a, in the second A_b. Y_1, r_a and r_b are stochastic.

1 Write down the budget equations for the consumer for the two periods.

2 The consumer's utility function is

$$U = E[u(C_0) + (1/(1+\rho))u(C_1)], \quad \rho > 0 \tag{1}$$

where

$$u(C) = \frac{1}{1-\theta}C^{1-\theta}, \quad \theta > 0$$

Derive the first-order conditions for maximum utility and give a verbal interpretation of them.

3 Assume (for this question only) that the two assets have the same expected return. Show that the first-order conditions in this case imply that

$$Cov(u'(C_1), 1 + r_a) = Cov(u'(C_1), 1 + r_b) \tag{2}$$

Which asset will be most in demand, a or b?

4 Suppose the consumer earns his income from farming and that one of the assets is shares in a food-processing firm that buys its raw materials from his and similar farms. Should he buy shares in this firm and how much?

5 Assume now that asset a is risk-free. How does this change the first-order conditions?

6 Now, assume that the economy is inhabited by a large number of consumers identical to the one we have studied. Explain how the first-order conditions from question 5 can then be used to determine the expected excess return on the risky asset.