

ECON4310 Exercise 2

Due 6/9 2010

1. Effects of savings on interest rates in Solow model.

Use the standard Solow model from Lecture 2.

- (a) Draw the graph that shows how the steady state level of k , k_* , is determined. Use this to illustrate how a decrease in the savings rate affects k_* and what the time path from the old to the new steady state looks like. Describe (in words or with a graph with time on the horizontal axis) what the time path of the real interest rate r will look like.
- (b) Use the steady state condition to derive an analytic expression for the effect of s on k_* . Determine the sign of the effect.

2. Depreciation in the Solow model

In the standard Solow model we neglect depreciation of the capital stock. Now, suppose capital depreciates from period to period by a factor δ . The accumulation equation for capital then becomes:

$$k_{t+1}(1 + \gamma) - k_t(1 - \delta) = sf(k_t)$$

where s should be interpreted as the *gross* savings rate.

- (a) Explain how the steady state value of k_t is determined.
- (b) Does a steady state with strictly positive output always exist?
- (c) Is the steady state stable?

3. Ramsey model with log utility

Suppose a social planner wants to maximize

$$\sum_{t=0}^{\infty} \beta^t \ln c_t \tag{1}$$

given

$$c_t = k_t^\alpha + k_t - k_{t+1}, \quad t = 0, 1, 2, \dots \tag{2}$$

and given $k_0 = \bar{k}_0$ and $k_t \geq 0$ for $t = 1, 2, \dots$. The natural growth rate is zero, while $0 < \beta < 1$ and $0 < \alpha < 1$.

- (a) Derive the first-order conditions for optimum.

- (b) What determines whether consumption will be growing or declining over time?
- (c) How is the steady state capital intensity in this economy determined? Explain in words why capital accumulation stops before the marginal productivity of capital is zero.
- (d) What is the savings rate in the steady state?
- (e) Suppose $\alpha = 0.3$ and $\beta = 0.96$. What are then the steady state levels of k and y ? How much would they differ if the social planner were more patient and had $\beta = 0.98$?