

## ECON4310 Exercise 4

Due 26/9 2011

### CRRA utility and consumption choice over two periods

The CRRA utility function is:

$$u(c) = \begin{cases} \frac{1}{1-\theta} c^{1-\theta} & \text{if } \theta > 0, \theta \neq 1 \\ \ln c & \text{if } \theta = 1 \end{cases}$$

1. Confirm that  $u'(c) > 0$  and  $u''(c) < 0$ .
2. What happens to  $u'(c)$  when  $c \rightarrow 0$  and when  $c \rightarrow \infty$ ?
3. Look at a consumer who lives for two periods (1 and 2). His utility function is

$$V = u(c_1) + \frac{1}{1+\rho} u(c_2)$$

where  $u$  is CRRA and  $\rho \geq 0$ . Explain what the parameters  $\rho$  and  $\theta$  (or  $\sigma = 1/\theta$  if you prefer) mean in terms of the consumer's preferences.

4. The consumer works in the first period but not in the second. He neither receives nor leaves any bequest. Hence, his budget constraint is

$$c_1 + \frac{1}{1+r} c_2 = w$$

where  $w$  is wage income. Derive the first-order condition for maximum utility.

5. Find an explicit solution for consumption in period 1. Discuss how this depends on wage income and on the interest rate. What roles do the parameters  $\rho$  and  $\theta$  (or  $\sigma$ ) play?