ECON4310 Fall 2011 Exercise 5

Due October 6

1 Government in steady state

Start from the standard overlapping-generations model with logarithmic utility, Cobb-Douglas production and no government. Population is growing with rate n and productivity with rate g. Each young individual supplies one unit of labor, while the old do not work.

1. We now introduce a government that collects two taxes. One tax is a proportional tax with rate τ on labor income, the other a proportional tax with rate θ on consumption. Modify the budget constraint of the consumers to include these two taxes. Find the utility-maximizing level of consumption when young.

2. Define the savings rate s as the ratio between a young consumer's savings and his *gross* wage income. How does the savings rate of a young consumer depend on the two tax rates? Why are the effects so different?

3. We want to look at a steady state where government consumption, tax revenues and government debt are constant when measured relative to the output level of the economy. Explain how you can find the tax ratio that makes the debt ratio stay constant. What combinations of τ and θ are needed to yield this tax level?

4. Let the level of government debt per efficiency unit of labor be constant at the level b. The capital stock per efficiency unit of labor in steady state, k_*, is then determined by the the equation:

$$k_* + b = \frac{s}{(1+n)(1+g)}(1-\alpha)k_*^{\alpha} \tag{1}$$

   Interpret this equation and explain briefly where it comes from.

5. Assume the economy is dynamically efficient. Compare the effect on k_* of an increase in b when the necessary interest payment to keep b constant are financed by the consumption tax and the tax on wage income.

2 Zero substitution elasticity in an overlapping generations model

Consumers live for two periods, supply one unit of labor in the first and retire in the second. In this exercise we assume that they always plan to have the same consumption in the two periods. Let $c_{y,t}$ and $c_{o,t}$ be the consumption of respectively a young and old person in period t, w_t the wage in period t and r_{t+1} the interest rate from period t to $t+1$. Let the assets that a young consumer carries over from period t to $t+1$ (the savings of the young in period t) be a_{t+1}.

There is no technological progress in the economy. The population grows with a rate n, which means that $L_t = (1+n)L_{t-1}$ where L_t is the number of young in period t. The production function is Cobb-Douglas,

$$y_t = f(k_t) = k_t^\alpha$$

where y_t is output per worker and k_t is the capital stock per worker. There is no depreciation.

1. Write down the budget constraint for a young consumer. How does her consumption level depend on the wage rate and the interest rate? Derive an expression for her savings rate. How does the savings rate depend on the interest rate?

2. Explain how the producers in this economy adapt to r_t and w_t.

3. The difference equation that describes how k_{t+1} follows from k_t can be written as
$$k_{t+1}(1+n) = \frac{1}{2+r_{t+1}}(1-\alpha)k_t^\alpha \tag{1}$$
Derive this equation.

4. Draw a graph with k_{t+1} on the horizontal and r_{t+1} on the vertical axis. Draw two curves in it: One that represents the firms' demand for capital in period $t+1$, and one that represents the savers' supply of capital as it follows from equation (1) for a given k_t. Which curve is the most elastic? Comment on the possibility for multiple equilibria.

5. Use the graph to look at the effect on k_{t+1} and r_{t+1} of a higher k_t.