Exercises to Seminar 4
ECON 4310

October 14, 2013

1 The RBC model

In this problem we will work through a simplified RBC model where we, as in the example covered in Lecture 11, will get an analytical solution. In the model we have a social planner's problem described by:

$$\begin{aligned}
\max_{\{c_s, n_s, k_{s+1}\}_{s=t}^{\infty}} & E_t \sum_{s=t}^{\infty} \beta^{s-t}[\log c_s + \phi \log(1 - n_s)] \\
s.t. & \\
c_t + k_{t+1} &= A_t k_t^{\alpha} n_t^{1-\alpha} + (1 - \delta) k_t \\
A_t &= A e^{z_t} \\
z_t &= \rho z_{t-1} + \varepsilon_t \\
c_t &\geq 0 \\
k_{t+1} &\geq 0 \\
0 \leq n_t &\leq 1
\end{aligned}$$

with $k_t > 0$ given. Can as before simplify by ignoring the conditions on n, c and k under 'normal' assumptions. We also simplify the model by assuming:

$$\delta = 1$$

1. Derive the first-order conditions with resepect to c_s, n_s and k_{s+1}.

2. Combine the first-order conditions for c_t and k_{t+1} to find the *inter-temporal* (Euler) condition:

$$1 = \beta E_t \left(\alpha A_{t+1} \left(\frac{k_{t+1}}{n_{t+1}} \right)^{\alpha - 1} \frac{c_t}{c_{t+1}} \right)$$

3. Show that you can combine the first-order conditions for c_t and n_t to find the *intratemporal* condition:

$$\phi \frac{c_t}{1-n_t} = (1-\alpha) A_t \left(\frac{k_t}{n_t} \right)^{\alpha}$$

4. A solution must make sure that the intra- and intertemporal conditions and the resource constraint are satisfied. Let us conjecture a solution of the form:

$$\begin{aligned} n_t &= \bar{n} \\ c_t &= \gamma_c A_t k_t^{\alpha} \bar{n}^{1-\alpha} \\ k_{t+1} &= \gamma_k A_t k_t^{\alpha} \bar{n}^{1-\alpha} \end{aligned}$$

   (a) Insert the solutions for c_t, c_{t+1} and n_t in the Euler equation. Show that it gives

   $$1 = \beta E_t \left(\alpha \frac{A_t k_t^{\alpha} \bar{n}^{1-\alpha}}{k_{t+1}} \right)$$

   Then insert the solution for k_{t+1}. Verify that γ_k must satisfy

   $$\gamma_k = \alpha\beta$$

   (b) Next, use the resource constraint to find

   $$\gamma_c = 1 - \gamma_k$$

   (c) Finally use the intratemporal condition to confirm that the constant value of labor supply is

   $$\bar{n} = \frac{1-\alpha}{1-\alpha+\phi(1-\gamma_k)}$$

5. We want to look at the impulse-response function for a shock to productivity. To find the impulse-response function we:

   - Start out in steady state
   - Shock productivity in period t (i.e. $\varepsilon_t = \Delta$) and keep the shocks equal to zero in future periods ($\varepsilon_{t+i} = 0$ for $i = 1, 2, \ldots$).
   - Then plot the response of k_{t+1+i}, c_{t+i} and n_{t+i} for $i = 0, 1, 2, 3, \ldots, k$. (Often it is more interesting to look at percentage-changes, but let us for simplicity look at levels now)

Assume $A = 1$, $\phi = 1$, $\beta = 0.99$, $\alpha = 0.33$, $\rho = 0.95$ and use $\Delta = 1$. Use Excel or some other software to draw the impulse-response functions.

6. For a variable x we let $\hat{x}_t$ denote its percentage deviation from steady state, i.e. $\hat{x}_t = (x_t - x^*)/x^*$. Find $\hat{c}_t$ and $\hat{k}_{t+1}$. (*Hint: You should get* $\hat{c}_t = \hat{k}_{t+1}$.

7. Compare the percentage increases in investment and consumption that we get for this model, with the pattern we saw in Lecture 13. Find one important reason for why the pattern is so different.

2 Labor supply

Let us look more carefully at the labor supply decision. For this problem we consider a more general utility function:

$$u(c, 1-n) = \log c + \phi \frac{(1-n)^{1-\theta} - 1}{1-\theta}$$

1. Try to show that

$$\lim_{\theta \to 1} \frac{(1-n)^{1-\theta} - 1}{1-\theta} = \log(1-n)$$

   Hint: Use L'Hopital's rule.

2. Update the intratemporal condition to the case with the more general utility function. Let us define the Frisch elasticity of leisure as the elasticity of $1-n$ with respect to the wage rate, holding the marginal utility of consumption constant. Use the intratemporal condition to verify that the Frisch elasticity of leisure is given by $-1/\theta$.

3. Then try to find the Frisch elasticity of *labor supply* using the same condition. Verify that it is not constant.

4. Use your answers to the last two questions to say something about the difference between utility functions defined in terms of leisure (such as the one above) and utility functions defined in terms of labor supply, such as

$$u(c, n) = \log c - \phi \frac{n^{1+\theta}}{1+\theta}$$

   (This question should help you understand why the Frisch elasticity is sometimes defined in terms of leisure and other times in terms of labor supply.)

5. Finally, let us look at a problem where labor supply is a choice along both the extensive and intensive margin. We are looking at an economy consisting of *two* individuals: Nick and Adam. They are along most dimensions identical: Both have utility over consumption and labor supply given by:

$$u(c_i, n_i) = \frac{c_i^{1-\sigma}}{1-\sigma} - \phi \frac{n_i^{1+\theta}}{1+\theta}$$

where $i = N, A$ indicates Nick or Adam. Note that the parameters in the utility function are the same. It is a one-period model, and both are maximizing utility subject to a budget constraint:

$$c_i = wn_i + b_i$$

(notice that they face the same wage rate). b_i is the exogenous assets that they have available. The only *difference* between them is that Nick has a flexible job where he can choose his number of hours freely, subject to the constraint that $0 \leq n_N \leq 1$. Adam, on the other hand, can only choose to work full-time or not work at all, i.e. $n_A \in \{0, 1\}$. Make sure you understand that Nick's Frisch elasticity of labor supply is $1/\theta$.

(a) For Nick, we find labor supply in the usual way (assuming an interior solution). Use the budget constraint to insert for c_i in the utility funciton. Find the first-order condition (the intratemporal optimality condition).

(b) Then find the decision-rule for Adam's labor supply (define the wage rate w_A^* that makes Adam indifferent between working and not working).

(c) Calculate the (macro) Frisch elasticity of labor supply for $w < w_A^*$.

(d) Calculate the (macro) Frisch elasticity of labor supply for $w > w_A^*$.

(e) Explain (but do not calculate) what the Frisch elasticity is at the point when w is just below w_A^*.

(f) What is the lesson for the relation between micro and macro Frisch elasticities?