Exercise on the q model
ECON 4310

September 20, 2013

This problem is taken directly from Obstfeld and Rogoff (1996). It deals with the q model, and you can work on it in the lecture-free week (a solution is posted on the web by the end of the week).

1 A simple q model

Suppose that a firm facing a market interest rate $1 + r$ has a production function given by $Y_t = A_t F(K_t)$, where A is a productivity parameter and we treat labor input as fixed. The firm's objective function is to maximize the present discounted value of profits. However, the firm faces adjustment costs to changing its capital stock. Specifically, it must pay $\chi I^2/2$ in adjustment costs in any period where it invests (or disinvests) at a rate I (note the slight difference from the cost function in class). Thus the firm maximizes:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[A_s F(K_s) - I_s - \chi \frac{I_s^2}{2}\right]$$

subject to the capital accumulation equation:

$$K_{t+1} = K_t + I_t$$

Let q_t be the Lagrange multiplier for the capital accumulation equation. The first order conditions for the firm's maximization problem come from solving:

$$\max_{\{I_s, K_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[A_s F(K_s) - I_s - \chi \frac{I_s^2}{2} - q_s(K_{s+1} - K_s - I_s)\right]$$

1. Differentiate the firm's objective function with respect to I_s and K_s to find the first-order conditions characterizing efficient investment.

2. Show that the first-order conditions imply the following system in q and K:

$$K_{t+1} - K_t = \frac{q_t - 1}{\chi},$$
$$q_{t+1} - q_t = rq_t - A_{t+1}F'(K_t + \frac{q_t - 1}{\chi})$$

3. Assume that the productivity parameter is constant at A. Find the steady state. Does it depend on adjustment costs?

4. Find a first-order approximation of the system and draw the phase diagram of the system with q on the vertical axis and K on the horizontal axis.

5. Using your graph, show what happens when there is an unanticipated permanent rise in A to A'. Show the new steady state and the transition to the new steady state.

6. Now suppose that the system is initially in a steady state corresponding to the productivity level A, but the firm learns (by surprise) on date t that A will rise permanently to A' at some future date T. Show, using your graph, what happens at time t and thereafter.

7. In the model from class, we showed that marginal q equals average q. Is that true in this formulation?