# Solution to exercise on the $q$ model
# ECON 4310

September 20, 2013

Solution is (as the problem was) taken from Obsteld and Rogoff (1996).

1. Differentiating the firm's objective function with respect to $I_s$ and $K_s$, we get:

$$q_s = 1 + \chi I_s$$
$$q_s = \frac{1}{1+r}[A_{s+1}F'(K_{s+1}) + q_{s+1}]$$

2. Use the capital accumulation equation to insert for investment in the top equation:

$$K_{t+1} - K_t = \frac{q_t - 1}{\chi}$$

Then use this condition to insert for $K_{t+1}$ in the first-order condition with respect to capital:

$$q_{t+1} - q_t = rq_t - A_{t+1}F'(K_t + \frac{q_t - 1}{\chi})$$

3. As in the model from class, the steady state is given by $\bar{q} = 1$ and $AF_K(\bar{K}) = r$. The steady state is independent of the adjustement costs, which determine only the speed of transition to the steady state.

4. The phase diagram looks just like what is on p.32 from slide set #10, but the exact expression for the slope of the $\Delta q = 0$ schedule has changed. Based on the linear approximation, it is now:

$$\frac{dq}{dK}|_{\Delta q=0} = \frac{AF''_{KK}(\bar{K})}{r - \frac{AF''_{KK}(\bar{K})}{\chi}} < 0$$

5. Improved productivity shifts the $\Delta q = 0$-locus (and changes the slope, but that is not so important to capture in the graph). Will have the same effect as the case analyzed on p.36 in slide set #10. Because the initial capital stock is given, $q$ rises in the short run and investment surges. Over time, however, $q$ falls back to 1 and investment converges to zero as $\bar{K} \to \bar{K}'$.

6. The principle for analyzing anticipated shocks is the same as in class (slides 39-40). As long as period $T$ has not arrived, the old curves govern the dynamics. But in period $T$ (when productivity goes from $A$ to $A'$) the curves from question (5) become operative. Nonetheless, the firm will adjust in anticipation so as to smooth its investment costs. Between dates $t$ and $T$, $q$ and $K$ will therefore follow the original equations of motion (those involving $A$). Thus, $q$ jumps initially and, until date $T$, continues rising as capital is accumulated. The firm reaches the new saddle-path precisely at date $T$, and thereafter $q$ falls toward 1 and $K$ rises to its new steady state. Note that the figure in class was for an expected interest rate increase, so what you draw here should be the "mirror image".

7. Marginal $q$ does not equal average $q$ because the installation cost function in this exercise is not linear homogenous in $K$ and $I$. To see what this means, do the same math as in slides 44-45 to find that:

$$q_t K_{t+1} = V_t - \frac{1}{2}\chi \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} I_t^2$$

   So the average $q$, which we measure as $V_t/K_{t+1}$, is in fact higher than marginal $q$. This is because bigger firms have a bigger capital adjustment cost per unit of investment than smaller firms, so looking at their value per unit of capital gives an exaggerated impression of the value created by an additional unit of capital in the firm.