Investment: Tobin's q

Lecture 10, ECON 4310

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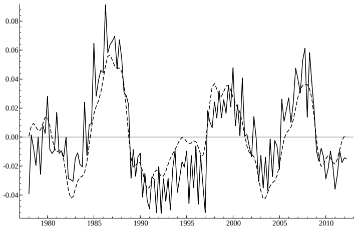
Why investment?

Today's lecture is devoted to the theory of investment. Why? Because investment behavior is an important determinant for:

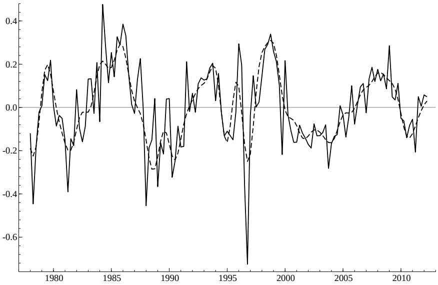
- Long run growth (through the role of capital)
- Business cycle dynamics

Investment over the business cycle

Investments are highly correlated with the business cycle and a lot more volatile than output. Here we have plots for the cyclical components of mainland GDP and oil investments for Norway (cyclical? Next lecture!). There are two lines because I have used two different measures of the cyclical component.



(a) Mainland GDP



(b) Oil investments

Investment over the business cycle II

The investment components are by far the most volatile time series (again: cyclical components).

Variable	Standard deviation
Mainland GDP	2.49
Consumption	2.38
Public consumption	1.58
Investments except oil	9.96
Residential investment	10.48
Oil investment	14.77
Exports (except oil sector)	4.50
Imports (except oil sector)	5.57

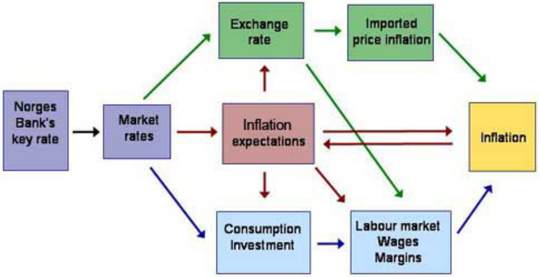
So: What determines the rate of investment?

In the very first macro models you were taught, investment plays an important role over the business cycle. A typical Keynesian investment function is:

$$I_t = a_0 + a_1 Y_t - a_2 r_t$$

where I_t is investment, Y_t is GDP and r_t is the real interest rate. So: Investment should be higher when the interest rate is low, and vice versa.

A similar view is found in typical central bankers' view of how monetary policy affects inflation (illustration from Norges Bank's website):



So: What determines the rate of investment? III

- However, ever since Haavelmo (1960) (*A Study in the Theory of Investment*), it has been recognized that the Keynesian investment function is inconsistent with the simple neoclassical model of investment.
- Moene and Rødseth (1991): Haavelmo's aim in this book was to "destroy the standard Keynesian demand function for investment", and "to offer an alternative".
- Main point: Neoclassical theory only predicts a relationship between the *desired stock of capital* and the interest rate. No reason to expect a smooth relationship between *investments* and the interest rate.

So: What determines the rate of investment? IV

Using a simple neoclassical model it is indeed possible to generate data series where the rate of investment is declining in the interest rate.

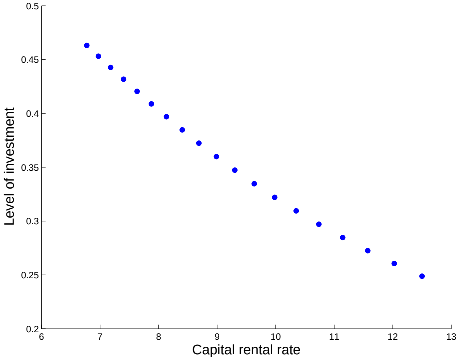


Figure: Scatter plot: Rate of investment as a function of the interest rate

So: What determines the rate of investment? V

But the previous figure was just a coincidence for the first 20 observations. When I generate longer time series we see that the true relationship is much less smooth:

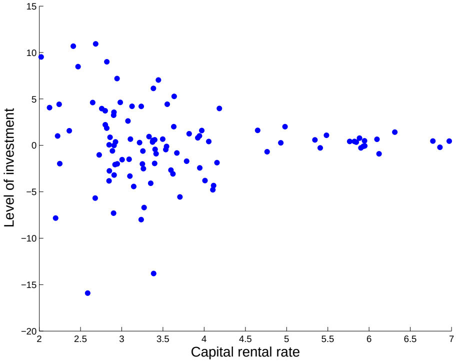


Figure: Scatter plot: Rate of investment as a function of the interest rate

So: What determines the rate of investment? VI

- The smooth plot was based on the first 20 observations where the interest rate was falling gradually from a high level and investment was rising slowly.
- After that the interest rate varies around a trend and investment rises and falls without any clear relationship to the interest rate level

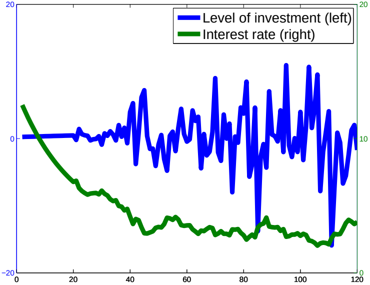


Figure: Time series behind the scatter plots

So: What determines the rate of investment? VII

Indeed, if we plot the rate of investment as a function of the *change* in the interest rate, it seems like we are much closer to an accurate description:

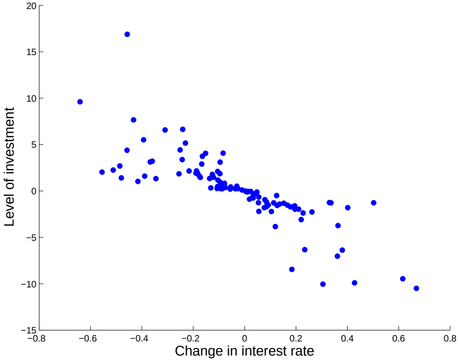


Figure: Scatter plot: Rate of investment as a function of the *change in* the interest rate

Outline

The value of a firm

Today we focus solely at firm's investment behavior. But firms are owned by households, and (at least as a starting point) we should assume that firm managers maximize the value of the firm for its owners.

- Let V_s denote the value of a firm at the end of period s
- Owners of the firm from the end of period s receive dividends d_{s+1} in the next period, and can sell the firm at a value V_{s+1}
- Assuming that there exists a risk free interest rate r, we know that the value in a perfect foresight case must satisfy:

$$1 + r = \frac{d_{s+1} + V_{s+1}}{V_s}$$

The value of a firm II

- Rewriting and then iterating forward we have:

$$V_t = \sum_{s=t+1}^{T} \left(\frac{1}{1+r}\right)^{s-t} d_s + \left(\frac{1}{1+r}\right)^{T} V_{t+T}$$

- The no-bubble condition is $\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T} V_{t+T} = 0$, and with that imposed we get:

$$V_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} d_s$$

$\Rightarrow$ The value of a firm reflects the NPV of future dividends.

- When making decisions, the firm manager should be concerned with maximizing the sum of dividends paid today plus the value at the end of period t: $V_t + d_t$, so the objective function should be:

$$d_t + V_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} d_s$$

Firm behavior

Lets assume a production function $A_t F(K_t, L_t)$. The firm hires labor but purchases its own capital (i.e. not rental market for capital). With no depreciation of capital, its profits in period t which are paid as dividends are:

$$d_t = A_t F(K_t, L_t) - w_t L_t - [K_{t+1} - K_t]$$

The firm manager thus chooses $\{L_s, K_{s+1}\}_{s=t}^{\infty}$ to maximize:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \{A_s F(K_s, L_s) - w_s L_s - [K_{s+1} - K_s]\}$$

taking K_t as given. First-order conditions:

$$A_s F_L'(K_s, L_s) = w_s$$

for $s \geq t$ and

$$A_s F_K'(K_s, L_s) = r$$

for $s > t$.

Firm behavior II

How does this give us a theory of investment? Well, since $I_t = K_{t+1} - K_t$, the rate of investment depends on what capital levels that come out of the first order conditions. Assume, for simplicity, that we are in a full employment equilibrium so $L_s = \bar{L}$ for all s. Then the first-order condition for labor just determines the real wage while

$$A_s F'_K(K_s, \bar{L}) = r$$

defines the optimal capital stock as a function of productivity and the interest rate, $K^*(A_s, r)$. The rate of investment is then:

$$I_s = K^*(A_{s+1}, r) - K^*(A_s, r)$$

Firm behavior III

To see how the interest rate affects investment we need to see how K^* changes:

$$\frac{dK^*}{dr} = \frac{r}{AF''_{KK}(K^*, \bar{L})}$$

which is negative under standard assumptions. We therefore have:

- Holding K_t constant, I_t will fall if r goes up since K_{t+1} falls
- But an increase in the interest rate level for all periods (or a change in the interest rate path) will affect both present and future capital! No reason to expect a smooth relationship between investment and the interest rate. Might be that investment can be both high and low for the same rate of interest.
- More important whether the interest rate is *rising* or *falling*, since that determines whether the capital stock is being *decreased* or *increased*.

This was one of the main points of Haavelmo (1960). Explains why the Keynesian investment function is inconsistent with a neoclassical theory of investment.

Problem: Infinite investment

- While this helps us understand the weakness of a Keynesian investment function, Haavelmo and others have pointed out that the basic neoclassical theory has a big problem as well.
- Best understood in a continuous time setup. First-order condition for capital is then basically the same:

$$A(s)F_K'(K(s),\bar{L}) = r$$

  while investment is $I(s) = \frac{dK(s)}{ds} = \dot{K}(s)$.
- By implicitly differentiating the first-order condition we find K^* as a smooth function of r.
- So discrete changes in the interest rate must lead to discrete changes in the capital stock

$\Rightarrow$ Discrete changes in the interest rate cause an infinite rate of investment!

- This is another argument against the Keynesian function: Investment cannot be a smooth function of the interest rate.
- But it is also a problem for the basic model: The rate of investment is clearly not infinite in practice

In a discrete time model we of course don't find infinite investment, but that is just camouflage.

Problem: Infinite investment II

- Put differently, the Keynesian investment function makes investment too smooth
- But the neoclassical model, although staring in the "right" place, predicts too rapid changes in investment.

$\Rightarrow$ Need to combine the neoclassical setup with a story for why investment is slower to adjust than in the baseline case

$\Rightarrow$ Natural solution: Add capital adjustment costs – which is what we will look at today

$\Rightarrow$ Haavelmo's solution: Build a model where investors are rationed in booms and invest zero in recessions

Outline

Capital adjustment costs

Will now present a model with adjustment cost based on the presentation in Obstfeld and Rogoff (Chapter 2.5, 1996). Romer's presentation is less suitable since it uses control theory. Advice for you: Learn the model as it is presented in this lecture, but use Romer's discussion of the model for improved understanding. (Only Romer chapter 8.1-8.6 (3rd edition) that is on the syllabus).

Capital adjustment costs II

In the baseline case the firm could purchase or sell capital with no cost other than the capital itself. Assume instead that capital is costly to adjust because of installation costs, production disruption, learning, etc. Profits in period s are given by:

$$A_s F(K_s, L_s) - w_s L_s - I_s - \frac{\chi}{2} \frac{I_s^2}{K_s}$$

where $I_s = K_{s+1} - K_s$. Adjustment costs are convex, so it is costly to adjust a lot at the time. Why relative to capital? Both intuitive and convenient for the analytical results.

Capital adjustment costs III

The firm now chooses capital and the rate of investment to maximize (we assume $L_s = \bar{L}$ for simplicity):

$$V_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[A_s F(K_s, \bar{L}) - w_s \bar{L} - I_s - \frac{\chi}{2} \frac{I_s^2}{K_s} \right]$$

subject to $K_{s+1} = K_s + I_s$. Let q_s be the current-valued Lagrange multiplier. Lagrangian expression:

$$L_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[A_s F(K_s, \bar{L}) - w_s \bar{L} - I_s - \frac{\chi}{2} \frac{I_s^2}{K_s} - q_s (K_{s+1} - K_s - I_s) \right]$$

First-order conditions:

$$I_s : \quad -\chi \frac{I_s}{K_s} - 1 + q_s = 0$$

$$K_{s+1} : \quad -q_s + \frac{1}{1+r} \left(A_{s+1} F_K(K_{s+1}, \bar{L}) + \frac{\chi}{2} \left(\frac{I_{s+1}}{K_{s+1}} \right)^2 + q_{s+1} \right) = 0$$

Capital adjustment costs IV

The Lagrange multiplier q plays a central role (yes, this is Tobin's q). As any other Lagrange multiplier, it is a shadow price. In this case, q_s is the shadow price of capital in place at the end of period s. From the first-order condition with respect to investment we see that:

$$q_s = 1 + \chi \frac{I_s}{K_s}$$

Under the optimal plan, the firm invests such that the marginal cost of an additional unit of capital (which equals 1 plus the adjustment cost) must equal the shadow price of capital. Can also write this as the investment equation that Tobin (1969) posited:

$$I_s = (q_s - 1)\frac{K_s}{\chi}$$

So investment is only positive when $q_s > 1$, i.e. when the shadow price of capital exceeds the price of new capital (before adjustments costs).

Capital adjustment costs V

Next consider the first-order condition with respect to future capital.

$$q_s = \frac{1}{1+r}\left(A_{s+1}F_K(K_{s+1},\bar{L}) + \frac{\chi}{2}\left(\frac{I_{s+1}}{K_{s+1}}\right)^2 + q_{s+1}\right)$$

This is like an investment Euler condition. The shadow price of capital today must equal the discounted value of

- the return of capital next period,
- what you save in adjustment costs next period
- the future shadow price (since capital can be sold next period).

Capital adjustment costs VI

In the same way as we used iterative substitution to rewrite the value of a firm, we can impose $\lim_{T\to\infty} \frac{q_{t+T}}{(1+r)^T} = 0$ (i.e. no bubble) and get:

$$q_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[A_s F_K(K_s, \bar{L}) + \frac{\chi}{2}\left(\frac{I_s}{K_s}\right)^2\right]$$

so q_t reflects the NPV of all future marginal return and reduced adjustment cost that you get from purchasing one unit of capital.

Phase diagram

OK: We have two first-order conditions as well as the constraint $I_t = K_{t+1} - K_t$. How to proceed? Use the first order condition for I_t to insert for investment in the two other equations. What we are left with is:

$$\Delta K_{t+1} = (q_t - 1)\frac{K_t}{\chi} \tag{1}$$

$$\Delta q_{t+1} = rq_t - AF_K(K_t(1 + \frac{q_t - 1}{\chi}), \bar{L}) - \frac{1}{2\chi}(q_{t+1} - 1)^2 \tag{2}$$

which we can use to draw a phase diagram for the dynamics of K and q. To do so we need:

- To know the steady state
- And then describe how K and q evolve away from steady state

Phase diagram II

The steady state is the case with $\Delta q = \Delta K = 0$:

$$0 = (\bar{q} - 1)\frac{\bar{K}}{\chi}$$

$$0 = r\bar{q} - AF_K(\bar{K}(1 + \frac{\bar{q} - 1}{\chi}), \bar{L}) - \frac{1}{2\chi}(\bar{q} - 1)^2$$

From the first equation it is clear that $\bar{q} = 1$. From the second we get that $\bar{K}$ must satisfy $AF_K(\bar{K}, \bar{L}) = r$.

Phase diagram III

Then we can use (1) and (2) to describe the dynamics away from steady state. However, the analysis is somewhat easier if we study a linear approximation of the system close to the steady state. The first-order approximation of (1) is:

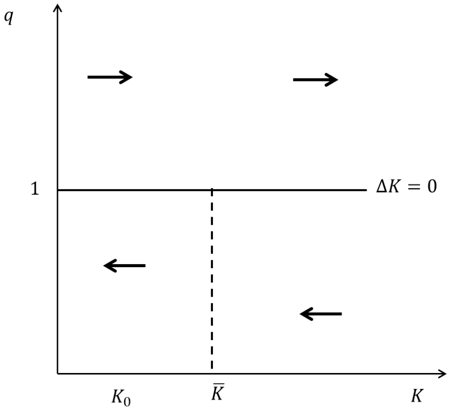
$$\Delta K_{t+1} = (q_t - 1)\frac{\bar{K}}{\chi} \tag{3}$$

while from (2) we get:

$$\Delta q_{t+1} = \left(r - \frac{A\bar{K}F_{KK}(\bar{K},\bar{L})}{\chi}\right)(q_t - 1) - AF_{KK}(\bar{K},\bar{L})(K_t - \bar{K}) \tag{4}$$

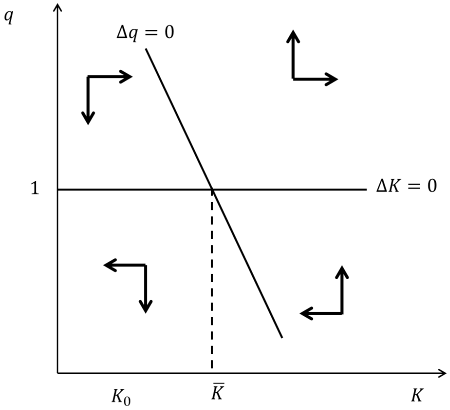
Phase diagram IV

Setting $\Delta K_{t+1} = 0$ in (3) a horizontal schedule at $q_t = 1$. For $q_t > 1$, the capital stock is growing and for $q_t < 1$ it is decreasing (because you invest only when $q > 1$).



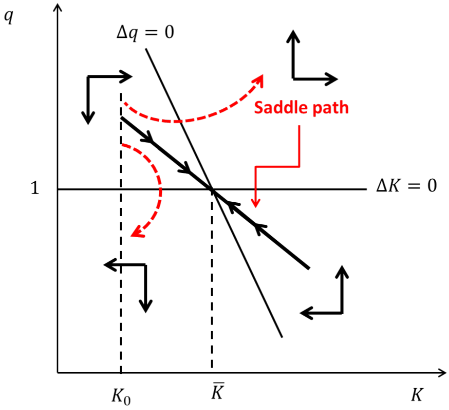
Phase diagram V

Setting $\Delta q_{t+1} = 0$ in (4) a downward sloping schedule for which (K, q) combinations that keep Tobin's q constant. For capital stocks to the left of the schedule the return to capital is high, so the price today grows relative to the future price (i.e. $\Delta q < 0$), so q is falling. To the right of the schedule the return on capital is so low that we need large capital gains to satisfy the optimality condition. q must grow.



Phase diagram VI

This system features saddle-path stability: For a given K_0 it is a unique level of q that puts the firm on a path that converges at the steady state.



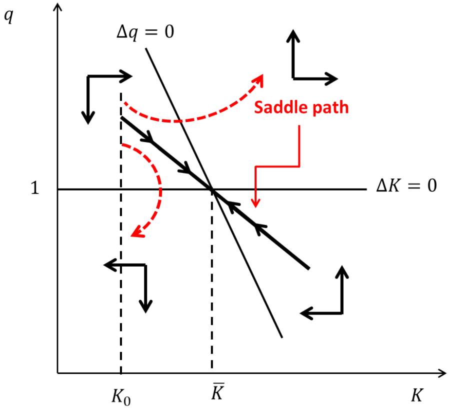
Phase diagram VII

- Starting from K_0, the firms capital stock is low and its return to capital is high
- Going straight to the steady state capital stock (as would have happened without adjustment costs) is too costly
- From (2) it is clear that such a situation leads to a high (above 1) q_t (and $\Delta q_{t+1} < 0$) since an extra unit of capital in this firm is valuable
- The high shadow value stimulates investment, so K grows
- As the capital stock is increasing, the shadow value falls, which then makes investment fall. In the end it converges to zero.

So having adjustment costs makes it possible to produce a more realistic investment response in which (i) the rate of investment is not infinite (even in continuous time), and (ii) that recognizes that it takes time to adjust the capital stock.

Phase diagram VIII

The unstable paths can be ruled out by assuming $\lim_{T\to\infty}(1+r)^{-T}q_{t+T} = 0$. In addition the one going to the south-west quadrant gives $q < 0$ and $K < 0$ at some point – which is not possible. Both paths are examples of *bubbles*; in the one case capital is increasing solely due to ever-increasing beliefs of q, while the other represents ever-decreasing beliefs of q.



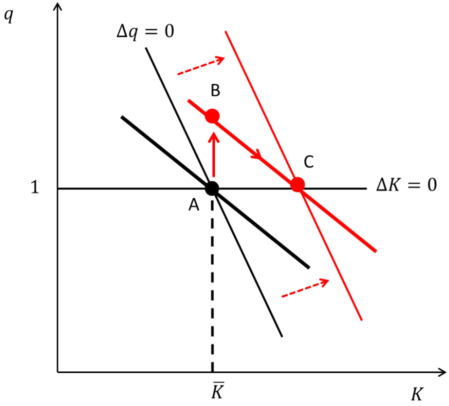
Investment and the interest rate

What happened to the interest rate? Recall that we were discussing the relationship between the interest rate and investment at the start of the lecture. We will now do three different experiments:

- A permanent reduction in r from r_0 to r_1
- A reduction in r believed to be permanent from r_0 to r_1 and then an increase to a level between r_1 and r_0
- An increase in r that is anticipated in advance

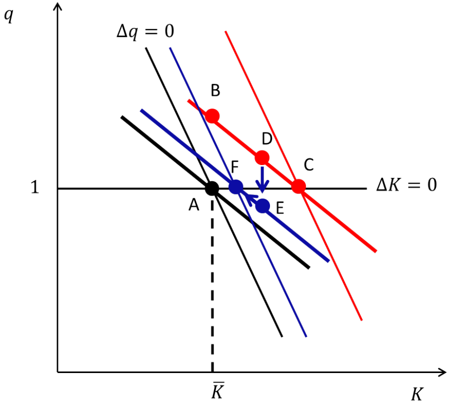
Investment and the interest rate II

First experiment: A permanent reduction in the interest rate. This affects the $\Delta q = 0$-locus. When the interest rate falls, a higher level of q is needed to keep $\Delta q = 0$, so the curve shifts up. There is a new saddle path. A jump in q makes us move from the old steady state (A) to the new saddle path (B). Will slowly converge to new steady state (C) with a higher capital stock.



Investment and the interest rate III

Second experiment: The same reduction as in experiment 1. But when the firm has come to point D, the interest rate is suddenly raised again to a level between the first and the second. Yet another saddle path, and we jump from (D) to (E). For this interest rate the capital stock is too large, so we get a period of disinvestment until we converge at (F).



Investment and the interest rate IV

- Experiment 1 shows that the model retains the intuitively plausible mechanism that a lower interest rate leads to higher investment. Further, it leads to a higher value of the firm, which is also intuitive.
- However, as experiment 2 shows, the relationship between investment and the interest rate is still far from as smooth as in the Keynesian investment function.

$\Rightarrow$ Point from Haavelmo still holds: There is no relationship between the rate of interest and the rate of investment

- We see that the movement from (E) to (F) gives disinvestment even though we can observe positive investment for a higher interest rate if we start out with $K < \bar{K}$.

Investment and the interest rate V

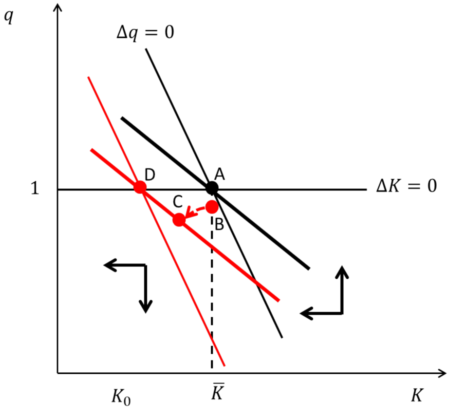
Third experiment: This time the firm anticipates a *higher* interest rate in the future. What happens? We know two things:

- A higher interest rate gives a new $\Delta q = 0$-locus and therefore also a new saddle path. When r changes, the new (K, q)-combination must be on the saddle path
- But we also know that (in this case perfect foresight but more general) a forward looking firm will not wait until r changes: it is only with the arrival of news that q jumps

This means that q will jump *before* the interest rate moves. Then it will be on a smooth path that ends up at the new saddle path exactly when r increases.

Investment and the interest rate VI

This is seen from the figure where we jump from (A) to (B) as soon as the news arrive. But since r hasn't changed yet the dynamics are still governed by the old system so we are on an unstable path. This takes us to point (C) exactly at the point in time when r increases. Afterwards we converge to D. Note that the value q "overshoots". Note also that the further ahead the change is expected, the smaller is the initial jump.



Outline

Average and marginal q

- q_t is the shadow price of capital held at the end of period t in the firm, so $q_t K_{t+1}$ is a natural way to value the firm. But q_t is unobservable.
- We have already derived the value of the firm V_t as the NPV of future dividends from the firm. This is presumably the stock-market value of the firm.
- Is there a relation between $q_t K_{t+1}$ and V_t?

Average and marginal q II

Assume first that there are no capital adjustment costs. Then $q_t = 1$.

- Since it is capital that makes up the firm, one should expect that the value of the firm equals its capital stock K_t
- But we also have that V_t equals the NPV of future dividends.
- These valuations are consistent since:

$$AF(K, L) = AF_K K + AF_L L = rK + wL$$

(Euler's theorem), so we can write dividends as:

$$d_s = [r_s K_s + w_s L_s] - w_s L_s - [K_{s+1} - K_s] = (1 + r)K_s - K_{s+1}$$

which makes the value of the firm:

$$V_t = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [(1 + r)K_s - K_{s+1}] = K_{t+1}$$

Average and marginal q III

Then we have adjustment costs. We start with (2):

$$q_t = \frac{1}{1+r}\left(A_{t+1}F_K(K_{t+1},\bar{L}) + \frac{\chi}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + q_{t+1}\right)$$

Multiply both sides by K_{t+1} and use $K_{t+1} = K_{t+2} - I_{t+1}$ for the last term to the right:

$$q_t K_{t+1} = \frac{1}{1+r}\left(A_{t+1}F_K(K_{t+1},\bar{L})K_{t+1} + \frac{\chi}{2}\frac{I_{t+1}^2}{K_{t+1}} - q_{t+1}I_{t+1} + q_{t+1}K_{t+2}\right)$$

Insert for $q_{t+1} = 1 + \chi(I_{t+1}/K_{t+1})$:

$$q_t K_{t+1} = \frac{1}{1+r}\left(A_{t+1}F_K(K_{t+1},\bar{L})K_{t+1} - \frac{\chi}{2}\frac{I_{t+1}^2}{K_{t+1}} - I_{t+1} + q_{t+1}K_{t+2}\right)$$

Do a forward iteration and impose $\lim_{T\to\infty}(1+r)^{-T}q_{t+T}K_{t+T+1} = 0$:

$$q_t K_{t+1} = \sum_{s=t+1}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(A_s F_K(K_s,\bar{L})K_s - \frac{\chi}{2}\frac{I_s^2}{K_s} - I_s\right)$$

Average and marginal q IV

Finally use that the production function is homogeneous of degree one so

$$A_s F_K(K_s, \bar{L}) = A_s F(K_s, \bar{L}) - w_s \bar{L}$$

which means:

$$q_t K_{t+1} = \sum_{s=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left(A_s F(K_s, \bar{L}) - w_s \bar{L} - \frac{\chi}{2}\frac{I_s^2}{K_s} - I_s\right) = V_t$$

Perfect! So $q_t K_{t+1}$ equals the stock market value of the firm.

Average and marginal q V

- q_t is often referred to as *marginal* q and is unobservable
- V_t/K_{t+1} is the *average* q and can be measured
- But unfortunately only under a set of simplifying assumptions that average and marginal q coincide (see Hayashi (1982)).

Outline

Summary

- Keynesian investment function $I(r)$ is too naive
- Simple neoclassical model shows that there is no simple relationship between investment and the rate of investment
- But the simple neoclassical model makes the capital stock adjust too quickly
- Capital adjustment costs makes the response more realistic
- But Haavelmo's critique of the Keynesian investment function continues to hold
- Tobin's model not only makes investment more realistic: it also gives us a link between investment and the shadow value of capital in the firm q
- Under simplifying assumptions, q can be identified from the stock market value per unit of capital