Search and unemployment

Lecture 16, ECON 4310

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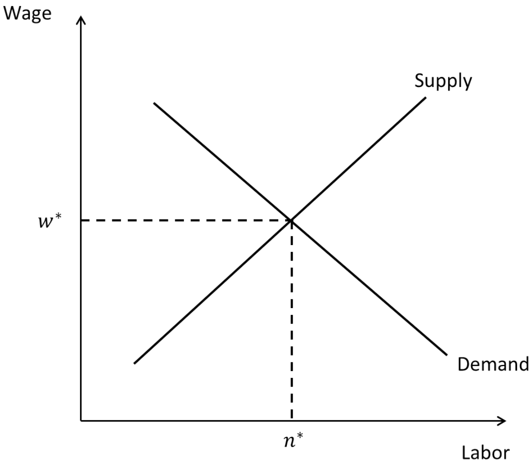
Introducing unemployment

Completely absent from our discussion for 15 lectures: Unemployment!

- Labor supply is an important part of our RBC model, but the representative agent always gets to work the optimal number of hours
- A model with labor lotteries does not change that: The probability of work is set to maximize the expected utility of each member in the group. No involuntary unemployment (perfect risk sharing, so unemployed are actually better off ex post!)
- But unemployment seems to be an important real-life issue. How to add?

Introducing unemployment II

Obivously, the usual supply and demand thinking does not allow for unemployment. At the market clearing wage, w^*, employment equals labor supply so there is no unemployment.

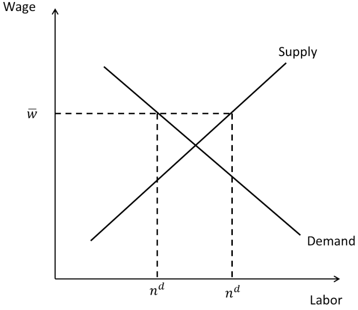


Introducing unemployment III

We therefore need some kind of labor market *frictions* to explain unemployment.

Introducing unemployment IV

Wage rigidity (stickiness) is a simple way to get unemployment:



Here unemployment can be measured as $n^s - n^d$ (and if total labor stock $= 1$ this is also the unemployment rate).

Introducing unemployment V

But we also know that the labor market is quite special. Maybe it would be fruitful to model more of these features? **Search theory** is one alternative.

Today's lecture

- Short intro to search theory
- One-sided search model
- If time: Two-sided search

Search theory

- Peter Diamond, Dale Mortensen and Chirsopther Pissarides won the Sveriges Riksbank Prize in 2010 "*for their analysis of markets with search frictions*"
- More from the press release: *Since the search process requires time and resources, it creates frictions in the market. On such search markets, the demands of some buyers will not be met, while some sellers cannot sell as much as they would wish. Simultaneously, there are both job vacancies and unemployment on the labor market.*
- Search theory can be applied to other fields as well (e.g. housing economics)

Search theory II

Unemployment arises in search models because you have

- Workers that would be willing to work if they found the 'right' job
- But they will keep turning down job offers that are not good enough
- Unemployment is an *equilibrium phenomenon*: All workers and firms are maximizing utility/profits
- No need to have wage rigidties: Search frictions are sufficient

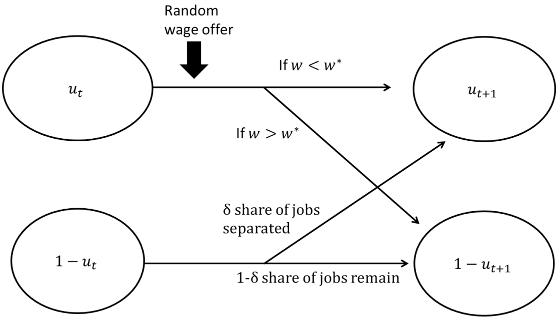
One-sided search

First we look at a 'one-sided' search model. We will ignore firms, and only focus at workers that are searching for a job. Basic mechanism:

- Wage offers are random
- Every worker has some reservation wage w^* that the offer must exceed if they are to accept
- Once accepted, they keep the job until it is destroyed (happens with an exogenous probability δ)

u_t: Unemployment rate. $1 - u_t$: Employment rate. Mass of workers normalized to unity.

One-sided search II: Simple illustration



One-sided search III

This picture illustrates the whole model. Let us put the story into equations. We need to

- Define the distribution of wage offers
- Characterize the reservation wage w^*

$\Rightarrow$ Makes it possible to pin down the steady state unemployment rate

One-sided search IV: Wage offers

We start with wage offers. Assume that wages are distributed on the interval $[0, \bar{w}]$, with cdf $F(w)$ and corresponding pdf $f(w)$.

One-sided search V: Reservation wage

Then we must characterize the reservation wage. That requires a lot more work. We want a micro-founded model, so we need to start out with some utility function and specify the optimization problem of the agent. Assume the following setup:

- There is a continuum of agents (of mass 1), each agent with a utility function $E_0 \sum_{t=0}^{\infty} \beta^t c_t$ [Note: linear utility]
- Labor supply is only on the extensive margin (0 or 1). We ignore disutililty from working.
- If unemployed, the agent recieves an unemployment benefit b
- If employed, the agent recieves a wage w
- There are no saving opportunities, so consumption in any period equals the benefit/wage you recieve

One-sided search VI: Reservation wage

- Agents start out as unemployed.
- Timing is that they first recieve unemployment benefit b. Then they recieve a job offer (the job starts in the next period).
- The wage they are offered is drawn from a distribution $F(w)$, with $w \in [0, \bar{w}]$ (so the 'quality' of the job offer is random).
- Assume that $\bar{w} > b$
- Must choose either to accept job offer or continue searching for another period

One-sided search VII: Reservation wage

- Once employed, you recieve w every period.
- But there is a constant probability of ‘job separation’, δ (job separation occurs at the *end* of a period)
- If separated, the agent enters the pool of unemployed and must start searching again

The choice we must analyze for an agent is therefore: Which wage offers should be accepted?

One-sided search VIII: Reservation wage

To answer this question, we will apply *value functions*, a concept from dynamic programming (DP). Necessary to make a short detour into the world of DP to know what this concept is.

Dynamic programming

Easiest to understand DP using a concrete example. Consider the social planner's problem in a simple Ramsey model:

$$\max_{\{c_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t.$$

$$c_t + k_{t+1} = Ak_t^{\alpha} + (1-\delta)k_t$$

Dynamic programming II

To understand DP, think about the problem as a *recursive* problem. For every period t there will be

- A state variable k_t (predetermined variables that we take as given)
- Two choice/control variables c_t and k_{t+1}

Once we think about it this way, we see that what the social planner is really doing, is to choose c_t and k_{t+1} in every period t, facing an identical optimization problem every time.

Dynamic programming III

What will the social planner maximize in period t?

- In period 0, we know that the utility flow from a certain choice of c_0 and k_1 is $u(c_0)$ subject to $c_0 + k_1 = Ak_0^\alpha + (1-\delta)k_0$.
- But you also know that your choice of k_1 affects future utility, since next periods state variable is k_1. Assume that future utility is given some function $v(k_1)$

Refer to $v(k)$ as the *value function*.

Dynamic programming IV

The period 0 maximization problem is therefore

$$\max_{c_0, k_1}[u(c_0) + \beta v(k_1)]$$

subject to $c_0 + k_1 = Ak_0^\alpha + (1-\delta)k_0$. If you know the function $v(k_1)$ it would now be normal procedure to find the solution for c_0 and k_1. These will be functions of k_0:

$$\begin{aligned} k_1 &= g(k_0) \\ c_1 &= Ak_0^\alpha + (1-\delta)k_0 - g(k_0) \end{aligned}$$

Dynamic programming V

Due to the recursive structure, we face a similar problem in period 1. Hence we see that $v(k_1)$ must satisfy

$$v(k_1) = \max_{c_1, k_2} [u(c_1) + \beta v(k_2)]$$

subject to $c_1 + k_2 = Ak_1^{\alpha} + (1 - \delta)k_1$. Aha!

Dynamic programming VI

We can state more generally:

$$v(k_t) = \max_{c_t, k_{t+1}} [u(c_t) + \beta v(k_{t+1})]$$

subject to $c_t + k_{t+1} = Ak_t^{\alpha} + (1 - \delta)k_t$. This is the **Bellman equation**. This is a *functional* equation, since it defines the unknown equation $v(k)$.

Dynamic programming VII

So there is one problem: The value function is generally not possible to find analytically. But in most cases we consider, the Bellman equation satisfies the contraction mapping theorem, which states that:

1. There is a unique value function that satisfies the Bellmann equation
2. If you start with some initial guess $v_0(k)$ and define

$$v_{i+1}(k) = \max_{c,k'}[u(c) + \beta v_0(k')]$$

subject to $c + k' = Ak^{\alpha} + (1-\delta)k$, then $\lim_{i\to\infty} v_{i+k} = v(k)$.

The first point secures that we can treat v as unique, and if we happen to find a funciton that satisfies the Bellman equation, then this is our value function. The second point tells us that we can use *value function iteration* to find the value function (numerical methods).

Dynamic programming VIII

- To work with search models, it is sufficient that you understand what a value function is, since all we do is to define them and use elegant transformations to solve the model
- One could also use DP to solve the RBC model, but we will not discuss that

One-sided search IX: Reservation wage

OK, we're back to the question of how to find the reservation wage. Let us define value functions as 'end of period' values, i.e. after the agent has decided whether to accept or reject an offer. In that case we have

- State variables: Employment status and wage offer accepted (if employed)

So the value function for an agent must in this case depend on two arguments: Employment status s and wage offer w.

$$\text{Value function} : v(s, w)$$

One-sided search X: Reservation wage

There are two possible values for employment status: $s \in \{u, e\}$ (refering to employed an unemployed). Furthermore, the wage accepted is only relevant for the employed, so $v(u, w) = v(u)$. To simplify notation let us therefore define

$$V_e(w) = v(e, w)$$
$$V_u = v(u)$$

We can state immediately that when an unemployed worker gets a wage offer w, it will accept if and only if $w \geq w^*$, where w^* is defined by

$$V_e(w^*) = V_u$$

This is the reservation wage.

One-sided search XI: Reservation wage

The value functions must be defined by Bellman equations. We start out with $V_e(w)$. A worker that is employed with a wage w at the end of a period, will for sure recieve w next period, and then face an exogenous risk of job separation. Hence:

$$V_e(w) = \beta\left[w + \delta V_u + (1-\delta)V_e(w)\right] \tag{1}$$

where the last term reflects the expected value of $V_e(w)$ at the end of next period.

One-sided search XII: Reservation wage

Next is V_u, the value function for an unemployed worker. We know that for sure the worker gets the unemployment benefit b next period. Then it might be that the worker recieves a large enough wage offer at the end of next period. This would make the worker change its employment status. Hence:

$$V_u = \beta \left[b + \int_0^{\bar{w}} \max \left\{ V_e(w), V_u \right\} f(w) dw \right] \tag{2}$$

where the integral represents the expected end-of-period value of V_u taking into account the 'risk' that it may change s from u to e.

One-sided search XIII: Reservation wage

We will now do a lot of transformations of the Bellman equations (1) and (2) to get an expression for w^* that we can interpret. For (2) we do the following:

$$\begin{aligned} V_u &= \beta \left[b + \int_0^{\bar{w}} \max\left\{V_e(w), V_u\right\} f(w) dw \right] \\ \Rightarrow (1+\rho) V_u &= b + \int_0^{\bar{w}} \max\left\{V_e(w), V_u\right\} f(w) dw \\ \Rightarrow \rho V_u &= b + \int_0^{\bar{w}} \max\left\{V_e(w) - V_u, 0\right\} f(w) dw \\ &= b + \int_{w^*}^{\bar{w}} \left(V_e(w) - V_u\right) f(w) dw \qquad (3) \end{aligned}$$

The last equality follows from the definition of w^*. LHS is the annuity value from being unemployed. RHS is the flow return (b) plus the expected net return from getting a job.

One-sided search XIV: Reservation wage

For (1) we do almost the same thing:

$$
\begin{aligned}
V_e(w) &= \beta \left[w + \delta V_u + (1-\delta) V_e(w) \right] \\
\Rightarrow (1+\rho) V_e(w) &= w + \delta V_u + (1-\delta) V_e(w) \\
\Rightarrow \rho V_e(w) &= w + \delta \left(V_u - V_e(w) \right)
\end{aligned}
$$

LHS gives the annuity value of employment at the wage w. RHS is the flow return from employment (w) plus the expected net loss from loosing your job. But let's not stop here. It will be useful to do rewrite the expression one extra time to get

$$
V_e(w) = \frac{w + \delta V_u}{r + \delta} \tag{4}
$$

One-sided search XV: Reservation wage

Then we use (4) to insert for $V_e(w)$ in (3):

$$\begin{aligned} \rho V_u &= b + \int_{w^*}^{\bar{w}} \left(V_e(w) - V_u\right) f(w) dw \\ &= b + \int_{w^*}^{\bar{w}} \left(\frac{w + \delta V_u}{r + \delta} - V_u\right) f(w) dw \\ &= b + \frac{1}{r + \delta} \int_{w^*}^{\bar{w}} \left(w - \rho V_u\right) f(w) dw \end{aligned} \tag{5}$$

One-sided search XVI: Reservation wage

(Almost) final step: Get rid off V_u by using the definition of w^*, namely

$$V_e(w^*) = V_u$$

which in (4) gives

$$\rho V_u = w^*$$

Taking this into (5) yields

$$w^* = b + \frac{1}{r+\delta} \int_{w^*}^{\bar{w}} (w - w^*) f(w) dw \tag{6}$$

Great! This equation involves only w^* and exogenous variables, so it determines the reservation wage.

One-sided search XVII: Reservation wage

But since we are having so much fun, let us continue with the algebra to get a simpler expression. What to do with this last integral? First, since w^* is a constant, it is clear that

$$\int_{w^*}^{\bar{w}} w^* f(w)dw = w^* [1 - F(w^*)]$$

Then for the rest of the integral we use that

$$\int_{w^*}^{\bar{w}} wf(w)dw = \Big|_{w^*}^{\bar{w}} wF(w) - \int_{w^*}^{\bar{w}} F(w)dw = \bar{w} - w^* F(w^*) - \int_{w^*}^{\bar{w}} F(w)dw$$

(integration by parts).

One-sided search XVIII: Reservation wage

Hence:

$$
\begin{aligned}
w^* &= b + \frac{1}{\rho+\delta} \int_{w^*}^{\bar{w}} (w - w^*) f(w) dw \\
&= b + \frac{1}{\rho+\delta} \left(\bar{w} - w^* F(w^*) - \int_{w^*}^{\bar{w}} F(w) dw - w^* \left[1 - F(w^*)\right] \right) \\
&= b + \frac{1}{\rho+\delta} \left(\bar{w} - w^* - \int_{w^*}^{\bar{w}} F(w) dw \right) \\
&= b + \frac{1}{\rho+\delta} \left(\int_{w^*}^{\bar{w}} \left[1 - F(w)\right] dw \right)
\end{aligned}
$$

One-sided search XIX: Reservation wage

What have we learned? We've learned that a utility-maximizing agent facing random job offers only accept wage offers greater than or equal to the reservation wage w^*, defined by

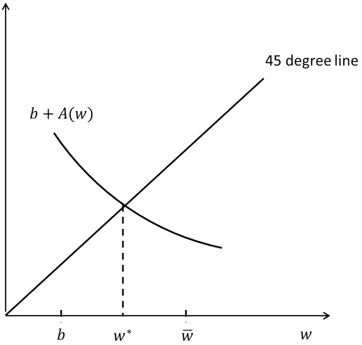
$$w^* = b + A(w^*) \tag{7}$$

where $A(w) = \frac{1}{\rho+\delta}\left(\int_w^{\bar{w}} [1 - F(x)]\, dx\right)$.

- $\Rightarrow$ Since the integral from $\bar{w}$ to $\bar{w}$ is zero, $A(\bar{w}) = 0$. Follows that (7) is not satisfied for $w^* = \bar{w}$ since we have assumed $b < \bar{w}$.
- $\Rightarrow$ Furthermore, $A(x) > 0$ for all $x \in [0, \bar{w})$ (since $F(x)$ is always ≤ 1) implying that (7) is not satisfied for $w^* = 0$
- $\Rightarrow$ By the same argument, we must have $w^* > b$

One-sided search XX: Reservation wage

Illustrated graphically:



Intuition for $w^* > b$: Unemployed workers will reject job offers with $w = b$ since they have the chance to recieve a higher wage in the future.

One-sided search XXI: Reservation wage

We can now do shift analysis to see what happens to w^* if b, $\bar{w}$, ρ, δ or $F(w)$ changes. Let us change b. Total differentiation in (7) gives:

$$\frac{dw^*}{db} = 1 - \frac{1}{\rho + \delta}[1 - F(w^*)]\frac{dw^*}{db}$$

or

$$\frac{dw^*}{db} = \frac{\rho + \delta}{\rho + \delta + 1 - F(w^*)} > 0$$

A higher benefit shifts $A(w)$ to the right in the diagram. Reservation wage goes up. The increase is dampened by the fact that a higher w^* reduces the probability that you get a wage offer that you accept in the future, but the net effect is still positive.

One-sided search XXII: Reservation wage

Changes in ρ and δ are the same. Total differentiation yields

$$\frac{dw^*}{d\rho} = \frac{dw^*}{d\delta} = -\frac{A(w^*)}{\rho + \delta + 1 - F(w^*)} < 0$$

- Higher ρ means that we discount the future more heavily (a lower β). This reduces the reservation wage since you are less willing to reject an offer and wait for something better.
- Higher δ increases the chances for job separation. Hence, this reduces the 'risk' that you are stuck in a relatively low-paying job, making you reduce your reservation wage.

One-sided search XXIII: Unemployment

Behavior of unemployment? We start out with some fraction of agents being unemployed, u_t. During one period we will observe that

- A share $1 - F(w^*)$ of the unemployed accepting job offers
- A share of δ of the emplyed being separated from their jobs

This gives the following law of motion for unemployment:

$$\begin{aligned} u_{t+1} &= u_t - u_t[1 - F(w^*) + (1 - u_t)\delta \\ &= u_t[F(w^*) - \delta] + \delta \end{aligned}$$

Steady state unemployment is found by setting $u_t = u_{t+1} = \bar{u}$:

$$\bar{u} = \frac{\delta}{\delta + 1 - F(w^*)}$$

One-sided search XXIV: Unemployment

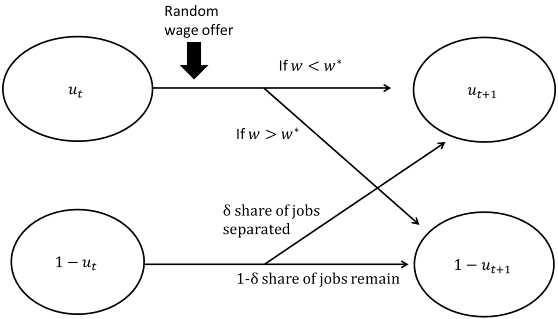
This is interesting. The model has a *steady state unemployment rate*, induced by the fact that the labor market has search frictions. Workers turn down job offers in equilibrium when $w < w^*$, even if $w > b$. The effect of increasing unemployment benefits, b, will be:

$$\begin{aligned}\frac{d\bar{u}}{db} &= -\frac{\delta}{(\delta + 1 - F(w^*))^2}\left(-F'(w^*)\frac{dw^*}{db}\right) \\ &= \frac{\bar{u}f(w^*)}{\delta + 1 - F(w^*)}\frac{dw^*}{db} > 0\end{aligned}$$

Higher unemployment benefits raise the steady state unemployment rate. Mechanism is simple: It raises the reservation wage, so more workers turn down wage offers.

One-sided search XXV

Fine. It is time to sum-up the one-sided search model. All we have done is to develop equations for this story:



One-sided search XXVI

By adding some structure, we defined the *value-functions* $V_e(w)$ and V_u using Bellman equations. Combined with defining w^* as the wage that solves $V_e(w^*) = V_u$, we manged to write the reservation wage as

$$w^* = b + A(w^*)$$

where $A(w) = \frac{1}{\rho+\delta}\left(\int_w^{\bar{w}} [1 - F(x)]\, dx\right)$. Furthermore, given w^*, the steady state unemployment rate is

$$\bar{u} = \frac{\delta}{\delta + 1 - F(w^*)}$$

One-sided search XXVII

What should be the next steps?

- We need a story for how wage offers come by

$\Rightarrow$ Hence natural to include *firms* that open vacancies

$\Rightarrow$ Takes us to two-sided search models where both firms and workers are searching