Search and unemployment (2nd lecture)

Lecture 17, ECON 4310

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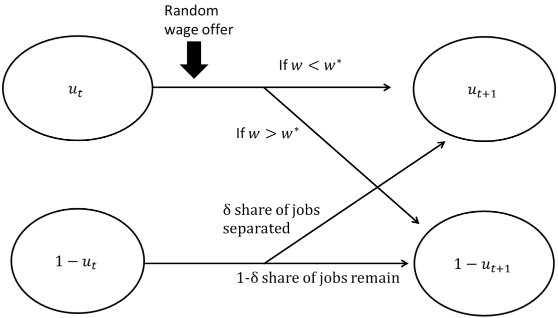
November 11, 2013

Last week: One-sided search

Structure of the model last week:



Two-sided search

Today: Two-sided search.

- We need a story for how wage offers come by

⇒ Natural to extend the model to include *firms* that open vacancies

- Let u_t denote the unemployment rate, while v_t is the *vacancy rate*
- Assuming a total mass of 1 among both workers and firms, then u and v also measure the number of unemployed and vacancies

Two-sided search II

Instead of having random job offers, we will now imagine that

- A certain number of unemployed, u, and vacant positions, v, lead to some number of matches, m [a match means that a worker is matched with a vacant position]
- Search frictions will affect how efficiently the economy can 'transform' unemployed and vacancies into matches

Two-sided search III

The matching technology will be conveniently/elegantly summarized in a *matching function*:

$$m_t = m(u_t, v_t)$$

where $m(u, v)$ is some constant-returns-to-scale function. The matching function summarizes all the search frictions that are present in the labor market. m_t measures the *number of matches*.

Two-sided search IV

Two definitions: Since there are u_t unemployed, the probability an unemployed is matched with a vacant position will be

$$\frac{m_t}{u_t} = \frac{m(u_t, v_t)}{u_t} = m(1, \frac{v_t}{u_t}) = m(1, \theta_t) = p(\theta)$$

where we refer to $\theta = v_t / u_t$ as a measure of *labor market tightness*. Similarly, the probability that a firm gets it vacant position filled is:

$$\frac{m_t}{v_t} = \frac{m(u_t, v_t)}{v_t} = m(\frac{u_t}{v_t},) = m(\frac{1}{\theta_t}, 1) = q(\theta)$$

Two-sided search V

We see that

$$p'(\theta) = m_2(1, \theta_t) > 0$$

so the probability of finding a job for an unemployed is increasing in labor market tightness. Conversely,

$$q'(\theta) = -m_1(\frac{1}{\theta}, 1)\frac{1}{\theta^2} < 0$$

showing that the probability that a firm gets it vacancy filled is decreasing in labor market tightness. It is normal to assume that $m(u, v)$ satisfies $\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} q(\theta) = 1$.

Two-sided search VI

It looks like the matching function has some empirical relevance:

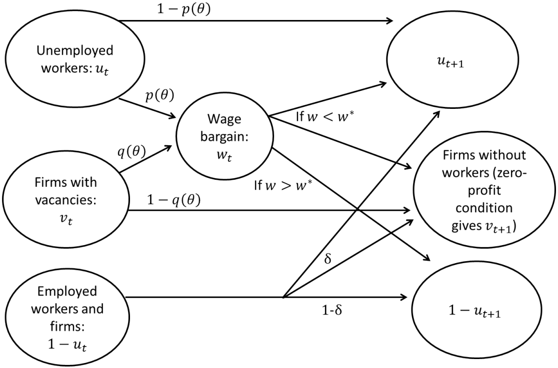
Figure : Empirical matching function [$p(\theta)$], 1951-2003. Source: Shimer (2005).

Two-sided search VII

- In the one-sided model we had 'search frictions' introduced through the short-cut of assuming a stochastic process for wage offers
- By modeling frictions explicitly using the matching funciton we are one step closer to a more proper model of the labor market

Two-sided search VIII

New structure:



Two-sided search IX

Steps needed to put this figure into equations:

1. Make assumptions regarding frictions
   - $\Rightarrow$ Matching function
2. Describe the wage bargain
   - $\Rightarrow$ Nash bargaining
3. Find the reservation wage
   - $\Rightarrow$ Will not be necessary given the wage bargaining
4. Determine the equilibrium number of vacancies
   - $\Rightarrow$ Zero-profit condition

Building model: Step 1

OK, so the first step is to specify that for

- u_t unemployed and v_t vacancies

$\Rightarrow$ We get $m_t = m(u_t, v_t)$ *matches*

A match means that one worker and a firm meet, and may initiate a worker-employer relationship if they agree on the wage.

Building model: Step 2

Next step: How do they bargain over the wage? This will be the main job, since we need to specify

- Preferences of workers and income alternatives
- Preferences of firms and production technology
- Value functions
- Wage bargaining procedure

Building model: Step 2 II

In the one-sided model workers recieved random wage offers. Now we assume that once it is matched with a firm:

- The worker gets employed (starting next period) if they agree on a wage w. If they don't agree, the worker gets an unemployment benefit b
- Separation happens with an exogenous probability δ
- Utility function is as in the one-sided model, $U = E_0 \sum_{t=0}^{\infty} \beta^t c_t$
- Still no saving opportunities (so consumption equals income every period)

Building model: Step 2 III

We can use what we learned in the one-sided model to say something about the value functions of the worker. The value function of an employed worker with a wage w has the same definition as before:

$$V_e(w) = \beta \left[w + \delta V_u + (1-\delta) V_e(w) \right]$$

which we once again re-write into

$$\rho V_e(w) = w + \delta \left(V_u - V_e(w) \right) \tag{1}$$

As for V_u, we treat it slightly different from before. To simplify, we will only consider the *steady state* where all workers have the same wage, w^{ss}. In that case the value function of an unemployed must satisfy

$$V_u = \beta \left[b + p(\theta) V_e(w^{ss}) + (1 - p(\theta)) V_u \right]$$

(so we don't need the integral-mess from the one-sided model) which we re-write into

$$\rho V_u = b + p(\theta)(V_e(w^{ss}) - V_u) \tag{2}$$

Building model: Step 2 V

Then we specify the preferences of the firm. Firms are assumed to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t - x_t)$$

where π_t are period t profits and x_t is the cost from posting vacancies. The firm 'consumes' all goods itself (workers do not recieve the profits). Their production technology is very simple:

- A firm that hires one worker will produce y units of output
- It produces nothing without a worker and cannot hire more than one
- It pays k to open a vacancy (which it may do if it is without a worker)
- Once a vacancy is opened, it is matched with a worker with probability $q(\theta)$

Building model: Step 2 VI

We also need to look at the value functions for the *firm*. First we define $J_e(y-w)$, the value function for a firm that has hired a worked at the wage w. This value function must satisfy

$$J_e(y-w) = \beta\left[y - w + (1-\delta)J_e(y-w) + \delta J_v\right]$$

where J_v is the value function of a firm that has opened a vacancy. We can write this condition as

$$\rho J_e(y-w) = y - w + \delta\left[J_v - J_e(y-w)\right] \tag{3}$$

Building model: Step 2 VII

What is the value of a vacancy? As for V_u, we only consider the steady state where all firms pay the same wage, w^{ss}. The value function must then satisfy

$$J_v = \beta \left[-k + q(\theta) J_e(w^{ss}) + (1 - q(\theta)) J_v \right]$$

where we again simplify by only looking at the option of hiring someone at the steady state wage rate. Rewrite this condition as

$$\rho J_v = -k + q(\theta) \left[J_e(w^{ss}) - V \right] \tag{4}$$

Building model: Step 2 VIII

With preferences specified [summarized by equations (1)-(4)], we can look at wage bargaining. For simplicity, we only look at the steady state where $u_t = u$ and $v_t = v$

- The *surplus* for a worker from a successful match is $V_e(w) - V_u$
- The *surplus* for a firm is $J_e(y - w) - J_v$

$\Rightarrow$ With Nash bargaining, the equilibrium wage maximizes the *Nash product*:

$$w = \arg\max_{w'} \left\{ V_e(w') - V_u \right\}^{\alpha} \left\{ J_e(y - w') - J_v \right\}^{1-\alpha}$$

subject to $V_e(w') - V_u \geq 0$ and $J_e(y - w') - J_v \geq 0$, where α is the bargaining weight of workers.

Assume an internal solution (ignore the non-neg constraints). We find the equilibrium wage by taking the first-order condition of the Nash product. It is:

$$\alpha V_e'(w)\left[J_e(y-w)-J_e\right]-(1-\alpha)J_e'(y-w)\left[V_e(w)-V_u\right]=0$$

The derivatives of V_e and J_e are found from equations (1) and (3):

$$V_e'(w)=J_e'(y-w)=\frac{1}{\rho+\delta}$$

which means that the first-order condition can be simplified into

$$\alpha\left[J_e(y-w)-J_e\right]=(1-\alpha)\left[V_e(w)-V_u\right]$$

Building model: Step 2 X

Now introduce

$$S = V_e(w) + J_e(y - w) - V_u - J_v \tag{5}$$

as the *total surplus* generated by the match. It follows from the first order condition that

$$V_e(w) - V_u = \alpha S \tag{6}$$

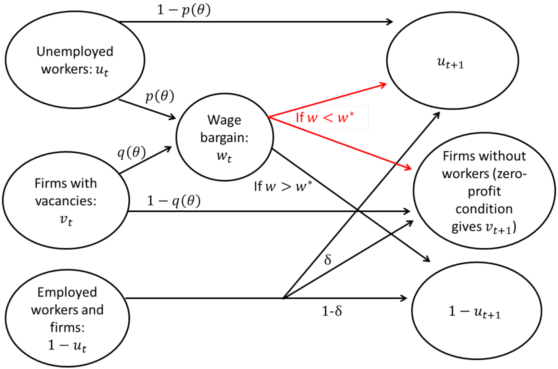
and

$$J_e(y - w) - J_v = (1 - \alpha)S \tag{7}$$

The solution to the Nash bargaining is a constant share of the total surplus to each side of the negotiation.

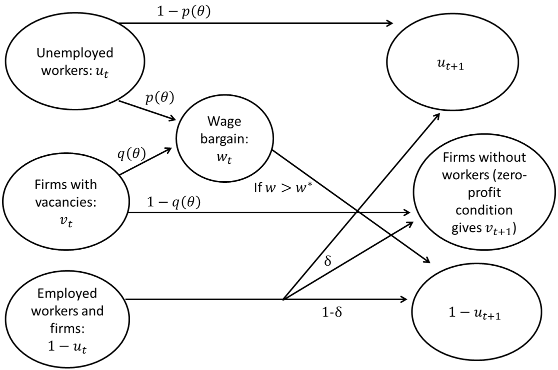
Building model: Step 3

Step 3 is to specify w^*. Howver, the Nash solution secures $V_e(w^{ss}) > V_u$ provided that $S > 0$ (to be confirmed later). Implication for w^*?



Building model: Step 3 II

w^* becomes 'irrelevant' since all wage offers are accepted in equilibrium.

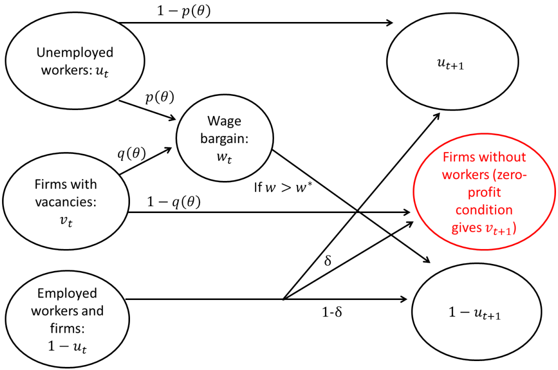


Building model: Step 4

After specifying preferences, obtaining value functions and deriving the solution to the Nash bargaining, we have 6 independent equations to determine 7 endogenous variables: $V_e(w)$, V_u, $J_e(y-w)$, J_v, S, w and θ.[1] Let us look at our figure to see where the last condition will come from. (This takes us to step 4)

[1] Equation (7) is not independent since it is implied by (5) and (6).

Building model: Step 4 II



Building model: Step 4 III

Aha! It is clear: We need some description of how much competition it is for workers on the firm side. We assume that new firms are always opening vacancies as long as it has a positive value. In equilibrium we therefore need

$$J_v = 0 \tag{8}$$

Building model: Solution

Great. We have completed the four steps from slide #11, and are ready to look at the solution. We will use equations (1)-(6) and (8) to solve for the endogenous variables. Summary of these equations:

$$\begin{aligned}
\rho V_e(w) &= w + \delta\left(V_u - V_e(w)\right) \\
\rho V_u &= b + p(\theta)(V_e(w^{ss}) - V_u) \\
\rho J_e(y-w) &= y - w + \delta\left[J_v - J_e(y-w)\right] \\
\rho J_v &= -k + q(\theta)\left[J_e(w^{ss}) - J_v\right] \\
S &= V_e(w) + J_e(y-w) - V_u - J_v \\
V_e(w) - V_u &= \alpha S \\
J_v &= 0
\end{aligned}$$

Trick for analyzing the model: Find the reduced form in terms of S and θ.

Building model: Solution II

First use (8) together with the (4):

$$0 = -k + q(\theta)\left[J_e(w^{ss}) - J_v\right]$$

Then insert for the Nash solution (equation (7)):

$$0 = -k + q(\theta)(1-\alpha)S$$

or

$$S = \frac{k}{(1-\alpha)q(\theta)} \qquad (*)$$

Building model: Solution III

Then start out with (5)

$$S = V_e(w) + J_e(y - w) - V_u - J_v$$

and insert for the value functions using (1)-(4) for $w = w^{ss}$:

$$\begin{aligned}
\Rightarrow \rho S =& w^{ss} + \delta \left[V_u - V_e(w^{ss})\right] + y - w^{ss} + \delta \left[J_v - J_e(y - w^{ss})\right] \\
& - b - p(\theta)[V_e(w^{ss}) - V_u] + k - q(\theta) \left[J_e(w^{ss}) - J_v\right] \\
=& y - b + k - \delta S - p(\theta)[V_e(w^{ss}) - V_u] - q(\theta)[J_e(w^{ss}) - J_v]
\end{aligned}$$

Building model: Solution IV

Use the implications of wage bargaining in (6) and (7):

$$\rho S = y - b + k - \delta S - p(\theta)\alpha S - q(\theta)(1-\alpha)S$$

Finally, from $(*)$, we have $k = q(\theta)(1-\alpha)S$, so that gives us

$$S = \frac{y-b}{\rho + \delta + \alpha p(\theta)} \qquad (**)$$

Building model: Solution V

These equations will determine the equlibrium value of S and θ:

$$S = \frac{k}{(1-\alpha)q(\theta)} \qquad (*)$$

$$S = \frac{y-b}{\rho+\delta+\alpha p(\theta)} \qquad (**)$$

while the remaining variables can be found from equations (1)-(8). Let us introduce the functions

$$G(\theta) = \frac{k}{(1-\alpha)q(\theta)}$$

and

$$F(\theta) = \frac{y-b}{\rho+\delta+\alpha p(\theta)}$$

Building model: Solution VI

- From $q'(\theta) < 0$, we see that $G'(\theta) > 0$
- Further, $\lim_{\theta \to 0} q(\theta) = 1$, so $\lim_{\theta \to 0} G(\theta) = \frac{k}{1-\alpha} > 0$

$\Rightarrow$ $G(\theta)$ starts at $k/(1-\alpha)$ and is forever increasing after that

- From $p'(\theta) > 0$, it follows that $F'(\theta) < 0$
- Further, $\lim_{\theta \to \infty} p(\theta) = 1$, so $\lim_{\theta \to 1} F(\theta) = \frac{y-b}{\rho+\delta+\alpha}$
- In addition, $p(0) = 0$, so $F(0) = \frac{y-b}{\rho+\delta}$

$\Rightarrow$ $F(\theta)$ starts at $(y-b)/(\rho+\delta)$ and will converge to $(y-b)/(\rho+\delta+\alpha)$. Between these points it is forever decreasing.

Building model: Solution VII

An equilibrium is secured as long as the two curves intersect for $\theta \in (0, \infty)$. This happens when

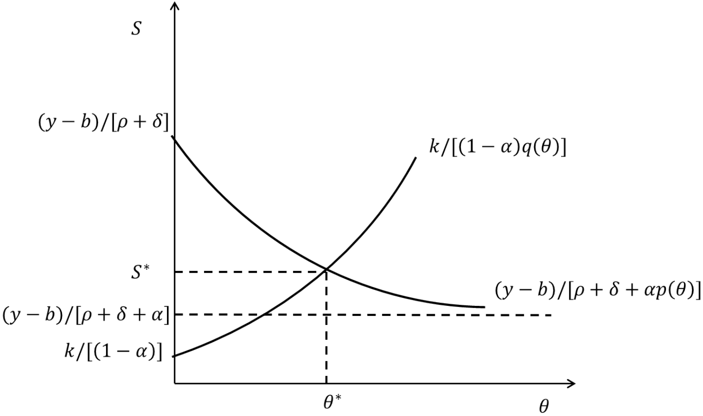
$$G(0) < F(0)$$

or

$$k < \frac{(1-\alpha)(y-b)}{\rho+\delta}$$

i.e. if the cost of vacancies is sufficiently small. The fact that the equilibrium value of S must lie between the (positive) values $G(0)$ and $F(0)$ also confirms that the surplus of a match is positive in equilibrium (which is why all matches lead to employment).

Building model: Solution VIII



Building model: Unemployment

What we have determined is the steady state labor tightness, θ. Last thing that remains: Look at flows in and out of unemployment (and vacancies).

- Every period a share $p(\theta)$ of the unemployed will find a job
- Exogenous separation rate: δ

Implies the following law of motion:

$$u_{t+1} = [1 - p(\theta)]u_t + \delta(1 - u_t)$$

and steady state level:

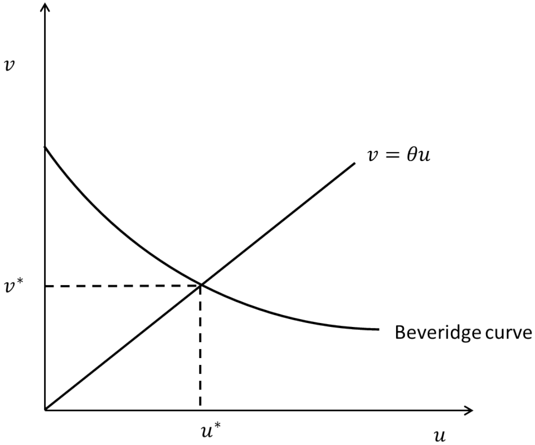
$$u^{ss} = \frac{\delta}{p(\theta) + \delta} \tag{9}$$

Since $\theta = v/u$, (9) gives the equilibrium relationship between u and v. We refer to it as the *Beveridge curve*. The steady state vacancy rate is simply

$$v^{ss} = \theta u^{ss}$$

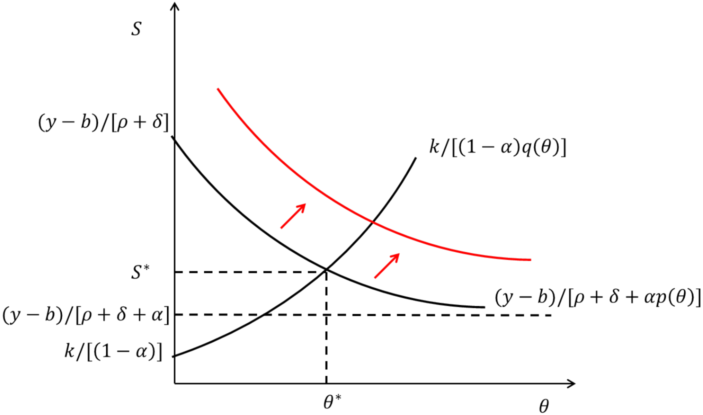
Building model: Unemployment II

Graphical illustration:



Comparative statics

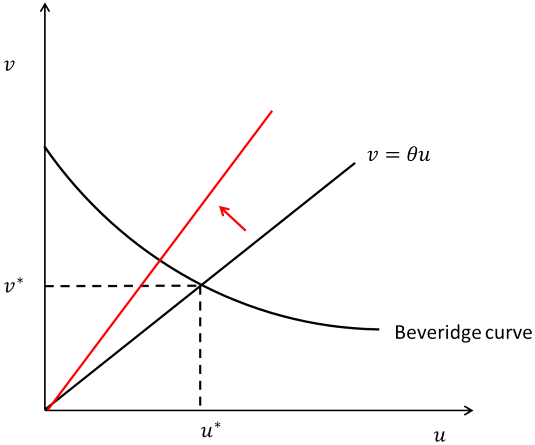
Consider now the effect on steady state values from shifts in the exogenous variables. An upward shift in $y - b$ will give:



i.e. increasing both S and θ in steady state.

Comparative statics II

Knowing that this increases θ, we can trace out the effects on unemployment and vacancies:



Comparative statics III

Or analytically:

$$\frac{du}{d\theta} = -\frac{\delta}{(\delta + p(\theta))^2} p'(\theta)$$
$$= -\frac{up'(\theta)}{\delta + p(\theta)} < 0$$

An increase in steady state tightness implies a lower unemployment rate.

Comparative statics IV

Conversely, the effect on vacancies is:

$$\begin{aligned}\frac{dv}{d\theta} &= u + \theta \frac{du}{dv} \\ &= u - \theta \frac{up'(\theta)}{\delta + p(\theta)} \\ &= u \left(\frac{\delta + p(\theta) - \theta p'(\theta)}{\delta + p(\theta)} \right)\end{aligned}$$

To find the sign here, use that $p(\theta) = m(1, \theta)$. It follows that

$$\theta p'(\theta) = \theta m_2(1, \theta) = m(1, \theta) - m_1(1, \theta)$$

since $m(u, v)$ is CRS. Thus:

$$\frac{dv}{d\theta} = u \left(\frac{\delta + m_1(1, \theta)}{\delta + p(\theta)} \right) > 0$$

Comparative statics V

So higher productivity will increase the steady state surplus of a match, as well as labor market tightness. Interpretation?

- Higher y directly increases firms' value from a match
- Due to Nash bargaining, this increase is shared between the worker and the firm (through higher wages)
- Firms start to post more vacancies since the value of a vacancy turns positive
- Higher v pushes up θ, which also implies a lower unemployment rate (since $p(\theta)$ goes up)

Same effects (but with opposite signs) operate if we increase b, except that the direct effect comes through the worker's surplus.

Important element of the model:

- Unemployment and vacancy ratios are likely to go in opposite directions (since θ raises v and lowers u).

$\Rightarrow$ Implies a so-called Beveridge curve

In our case we have only looked at differences in steady state, but can be shown to hold in regular business cycle models as well.

Beveridge curve II

Such a relationship is indeed an empirical regularity:

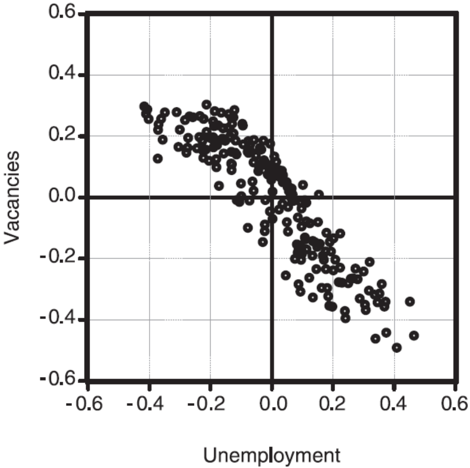


Figure : Quarterly US Beveridge curve, 1951-2003. Source: Shimer (2005).