How much should the nation save?

Econ 4310 Lecture 2

Asbjorn Rodseth

University of Oslo

August 21, 2013

Outline

- Golden Rule
- Social planner's choice
- Market solution
- Bringing back natural growth

Slide set covers first two, lecture may go further

The Golden Rule of Accumulation

Consumption per efficiency unit of labor in steady state is:

$$c = f(k) - \gamma k \tag{1}$$

First order condition for maximum is $f'(k) - \gamma = 0$. Golden rule level of k, k^{**} is determined by

$$f'(k^{**}) = \gamma \tag{2}$$

$$r^{**} = \gamma$$

Interest rate equal to natural growth rate
Savings rate required to reach k^{**}:

$$s^{**} = \gamma k^{**}/f(k^{**}) = r^{**}k^{**}/f(k^{**})$$

Along the Golden rule path the savings rate equals the income share of capital.
If s is increased beyond s^{**}, consumption is reduced both now and in all future!

The Ramsey model, simple version

Social planner maximizing utility over all future generations given

- Zero population growth
- zero productivity growth

Welfare function

$$U_0 = u(c_0) + \beta u(c_1) + \beta^2 u(c_2) + ... = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$0 < \beta < 1$ discount factor (subjective)

$\beta = \dfrac{1}{1+\rho}$ $\rho =$ discount rate, degree of impatience

$u(C)$ period utility $u' > 0$ $u'' < 0,$ $u'(0) = \infty$

The accumulation equation

$$y_t = f(k_t)$$

- y_t output
- k_t capital
- c_t consumption

$$k_{t+1} = k_t + i_t$$

$$k_{t+1} = k_t + f(k_t) - c_t \Leftrightarrow c_t = f(k_t) + k_t - k_{t+1} \tag{2}$$

The maximization problem

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

given

$$c_t = f(k_t) + k_t - k_{t+1} \text{ and } k_t \geq 0, \;\; k_0 = \bar{k_0}$$

Insert for c_t from accumulation equation in utility function.

Solution

Utility function after insertions is :

$$U_0 = \ldots + \beta^t u(f(k_t) + k_t - k_{t+1}) + \beta^{t+1} u(f(k_{t+1}) + k_{t+1} - k_{t+2}) + \ldots$$

$k_t + 1$ appears only in terms t and $t + 1$
Necessary condition for maximum are:

$$\frac{\partial U_0}{\partial k_{t+1}} = -\beta^t u'(c_t) + \beta^{t+1} u'(c_{t+1})(1 + f'(k_{t+1})) = 0$$

or

$$u'(c_t) = \beta u'(c_{t+1})(1 + f'(k_{t+1})) \text{ for } t = 0, 1, 2, \ldots, \infty \tag{3}$$

The Consumption Euler-equation

$$u'(c_t) = \beta u'(c_{t+1})(1 + f'(k_{t+1})) \text{ for } t = 0, 1, 2, \ldots, \infty \tag{4}$$

- A marginal transfer of resources between periods t and $t + 1$ should not increase total welfare
- A marginal transfer of resources between any two periods should not increase total welfare

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1}{1 + f'(k_{t+1})} \text{ for } t = 0, 1, 2, \ldots, \infty \tag{5}$$

- Marginal rate of substitution should equal marginal rate of transformation

Problem not solved yet

Two difference equations:

$$
\begin{aligned}
k_{t+1} &= f(k_t) + k_t - c_t & (6)\\
u'(c_{t+1}) &= [\beta(1+f'(k_{t+1}))]^{-1} u'(c_t) & (7)
\end{aligned}
$$

Non-linear system in two unknown time series. One initial condition (k_0 given)

- Exclude paths where $k_t \to 0$ when $t \to \infty$ (resource constraint).
- Choose the path with highest utility among the remaining

- Closed form solutions only in special cases. Simulation usually necessary.

The stationary state

Definition:

$$k_{t+1} = k_t = k^* \text{ and } c_{t+1} = c_t = c^*$$

Insert in difference equations (**??**) and (**??**):

$$\begin{aligned} k^* &= f(k^*) + k^* - c^* \\ u'(c^*) &= [\beta(1 + f'(k^*))]^{-1} u'(c^*) \end{aligned}$$

Solution:

$$\begin{aligned} f'(k_*) &= (1-\beta)/\beta = \rho && (8) \\ c^* &= f(k^*) && (9) \end{aligned}$$

- Subjective discount rate determines capital intensity. item Capital intensity determines output.
- No savings, consumption = output

Comparison to golden rule

Remember: No population growth, no productivity growth

- Golden rule: $f'(k^*) = 0$
- Ramsey rule: $f'(k^*) = \rho > 0$
- Social planner chooses less capital

Graphical solution

Phase diagram will be drawn to show:

- Social planner will choose a path that leads to the stationary state
- If$k_0 < k_*$,both consumption increases gradually as the economy moves towards the steady state.