Open vs. closed economies

Lecture 8, ECON 4310

Tord Krogh

September 11, 2013

Open economy models in general

Models of the open economy have always been a central part of macro.

Open economy models in general

Models of the open economy have always been a central part of macro.

- Traditional IS-LM type models for open economies. Mundell-Fleming-Tobin ('Old-school')

Open economy models in general

Models of the open economy have always been a central part of macro.

- Traditional IS-LM type models for open economies. Mundell-Fleming-Tobin ('Old-school')
- Dornbusch: Rational expectations model for exchange rates (Old-school + RE)

Open economy models in general

Models of the open economy have always been a central part of macro.

- Traditional IS-LM type models for open economies. Mundell-Fleming-Tobin ('Old-school')
- Dornbusch: Rational expectations model for exchange rates (Old-school + RE)
- Backus, Kehoe, Kydland: International RBC-models (Microfounded, all prices flexible)

Open economy models in general

Models of the open economy have always been a central part of macro.

- Traditional IS-LM type models for open economies. Mundell-Fleming-Tobin ('Old-school')
- Dornbusch: Rational expectations model for exchange rates (Old-school + RE)
- Backus, Kehoe, Kydland: International RBC-models (Microfounded, all prices flexible)
- Obstfeld and Rogoff: New Open Economy Macroeconomics (Microfounded, nominal frictions)

Open economy models in general

Models of the open economy have always been a central part of macro.

- Traditional IS-LM type models for open economies. Mundell-Fleming-Tobin ('Old-school')
- Dornbusch: Rational expectations model for exchange rates (Old-school + RE)
- Backus, Kehoe, Kydland: International RBC-models (Microfounded, all prices flexible)
- Obstfeld and Rogoff: New Open Economy Macroeconomics (Microfounded, nominal frictions)
- New Keynesian Open economy (NOEM ++)

Open economy models in general

Models of the open economy have always been a central part of macro.

- Traditional IS-LM type models for open economies. Mundell-Fleming-Tobin ('Old-school')
- Dornbusch: Rational expectations model for exchange rates (Old-school + RE)
- Backus, Kehoe, Kydland: International RBC-models (Microfounded, all prices flexible)
- Obstfeld and Rogoff: New Open Economy Macroeconomics (Microfounded, nominal frictions)
- New Keynesian Open economy (NOEM ++)

Today we focus at how the simple Ramsey-type and OLG models differ between closed and open economies. ECON 4330 goes more in-depth. Main reference: Rødseth's note (see the course website).

How does the open economy dimension matter?

How does the open economy dimension matter?

- 'Real models': Goods and international assets are traded across borders. Important baseline case: A frictionless international capital market.

How does the open economy dimension matter?

- 'Real models': Goods and international assets are traded across borders. Important baseline case: A frictionless international capital market.
    - World equilibrium models: Typically two-country models

How does the open economy dimension matter?

- ‘Real models’: Goods and international assets are traded across borders. Important baseline case: A frictionless international capital market.
    - World equilibrium models: Typically two-country models
    - Small open economy: Perspective where the outside world is infinitely large. Take world variables as exogenous.

How does the open economy dimension matter?

- 'Real models': Goods and international assets are traded across borders. Important baseline case: A frictionless international capital market.
  - World equilibrium models: Typically two-country models
  - Small open economy: Perspective where the outside world is infinitely large. Take world variables as exogenous.
- Nominal models: Exchange rate determination, pass-through, monetary policy spill-overs.

How does the open economy dimension matter?

- 'Real models': Goods and international assets are traded across borders. Important baseline case: A frictionless international capital market.
  - World equilibrium models: Typically two-country models
  - Small open economy: Perspective where the outside world is infinitely large. Take world variables as exogenous.
- Nominal models: Exchange rate determination, pass-through, monetary policy spill-overs.

The models we look at in this course are all 'real' – ECON 4325 deals with models with nominal frictions (for the closed economy), while ECON 4330 has both things.

Outline

Dynastic models

We start with a Ramsey-type model for a *small open economy*. Key change from closed economy: Country can borrow or lend unlimited amounts at the international capital market. Risk-free real rate of interest r^*.

Dynastic models

We start with a Ramsey-type model for a *small open economy*. Key change from closed economy: Country can borrow or lend unlimited amounts at the international capital market. Risk-free real rate of interest r^*.

$\Rightarrow$ Aggregate savings are thus no longer equal to aggregate real investment.

Dynastic models

We start with a Ramsey-type model for a *small open economy*. Key change from closed economy: Country can borrow or lend unlimited amounts at the international capital market. Risk-free real rate of interest r^*.

$\Rightarrow$ Aggregate savings are thus no longer equal to aggregate real investment.

$\Rightarrow$ Capital stock becomes independent of domestic savings preferences

Dynastic models

We start with a Ramsey-type model for a *small open economy*. Key change from closed economy: Country can borrow or lend unlimited amounts at the international capital market. Risk-free real rate of interest r^*.

- $\Rightarrow$ Aggregate savings are thus no longer equal to aggregate real investment.
- $\Rightarrow$ Capital stock becomes independent of domestic savings preferences
- !!! Key question: Under what conditions will there be a balanced growth path for the small open economy?

Dynastic models II

Aggregate production is standard:

$$Y_t = F(K_t, A_t L_t).$$

Dynastic models II

Aggregate production is standard:

$$Y_t = F(K_t, A_t L_t).$$

Productivity and labor input follow exogenous processes:

$$A_t = (1+g)^t A_0,$$
$$L_t = (1+n)^t L_0.$$

Dynastic models II

Aggregate production is standard:

$$Y_t = F(K_t, A_t L_t).$$

Productivity and labor input follow exogenous processes:

$$A_t = (1+g)^t A_0,$$
$$L_t = (1+n)^t L_0.$$

We define intensive form: $y_t = Y_t / A_t L_t$, $k_t = K_t / A_t L_t$, and if F is homo. of degree 1:

$$y_t = f(k_t).$$

Dynastic models III

In a competitive equilibrium where the representative firm is a price-taking profit maximizer, the marginal products of capital and labor equal their real factor prices. So:

$$\begin{aligned} f'(k_t) &= r_t, \\ f(k_t) - k_t f'(k_t) &= w_t, \end{aligned}$$

(for $w_t = W_t/A_t$).

Dynastic models III

In a competitive equilibrium where the representative firm is a price-taking profit maximizer, the marginal products of capital and labor equal their real factor prices. So:

$$f'(k_t) = r_t,$$
$$f(k_t) - k_t f'(k_t) = w_t,$$

(for $w_t = W_t/A_t$). But r_t is no longer a domestic price that is determined independently of the rest of the world. For a frictionless international capital market we need rates of return to be equalized. Hence:

$$r_t = r^*,$$

(where we for simplicity assume that the world interest rate is constant). Impact?

Dynastic models III

In a competitive equilibrium where the representative firm is a price-taking profit maximizer, the marginal products of capital and labor equal their real factor prices. So:

$$f'(k_t) = r_t,$$
$$f(k_t) - k_t f'(k_t) = w_t,$$

(for $w_t = W_t/A_t$). But r_t is no longer a domestic price that is determined independently of the rest of the world. For a frictionless international capital market we need rates of return to be equalized. Hence:

$$r_t = r^*,$$

(where we for simplicity assume that the world interest rate is constant). Impact? **The capital intensity and real wage are independent of domestic saving decisions!**

Dynastic models IV

If the world interest rate is constant, the capital intensity jumps to its steady state value in the initial period, and is constant from then on.

Dynastic models IV

If the world interest rate is constant, the capital intensity jumps to its steady state value in the initial period, and is constant from then on. Since we know the capital intensity we can also find the level of investment if we use:

$$K_{t+1} = K_t + I_t.$$

(For simplicity: No depreciation).

Dynastic models IV

If the world interest rate is constant, the capital intensity jumps to its steady state value in the initial period, and is constant from then on. Since we know the capital intensity we can also find the level of investment if we use:

$$K_{t+1} = K_t + I_t.$$

(For simplicity: No depreciation). Divide by $A_t L_t$:

$$(1+n)(1+g)k = k + i_t \Rightarrow i = \gamma k,$$

where $\gamma = n + g + ng$ is the natural growth rate. So for a constant world interest rate, we also find a constant level of investment (in intensive form), which equals the amount necessary to keep the capital intensity constant.

Dynastic models IV

If the world interest rate is constant, the capital intensity jumps to its steady state value in the initial period, and is constant from then on. Since we know the capital intensity we can also find the level of investment if we use:

$$K_{t+1} = K_t + I_t.$$

(For simplicity: No depreciation). Divide by $A_t L_t$:

$$(1+n)(1+g)k = k + i_t \Rightarrow i = \gamma k,$$

where $\gamma = n + g + ng$ is the natural growth rate. So for a constant world interest rate, we also find a constant level of investment (in intensive form), which equals the amount necessary to keep the capital intensity constant. **Again: In the small open economy, investment is separated from savings.**

Dynastic models V

Re-cap:

Dynastic models V

Re-cap:

- World interest rate determines capital intensity

Dynastic models V

Re-cap:

- World interest rate determines capital intensity
- Capital intensity determines real wage, investment and output (intensive form)

Dynastic models V

Re-cap:

- World interest rate determines capital intensity
- Capital intensity determines real wage, investment and output (intensive form)

What about consumption choices?

Dynastic models V

Re-capː

- World interest rate determines capital intensity
- Capital intensity determines real wage, investment and output (intensive form)

What about consumption choices? Will follow from utility maximization. Since output is independent of consumption choices, the consumption profile will pin down the *current account* and the evolution of *net foreign debt*.

Dynastic models VI

Simple accounting: b_t are net household assets, $b_{f,t}$ is net foreign assets. Since households own either capital or invest abroad, we have:

$$b_{f,t} = b_t - k_t$$

The net foreign asset position is determined solely by the difference between households' presumably optimal savings decisions and the optimal capital stock.

Dynastic models VI

Simple accounting: b_t are net household assets, $b_{f,t}$ is net foreign assets. Since households own either capital or invest abroad, we have:

$$b_{f,t} = b_t - k_t$$

The net foreign asset position is determined solely by the difference between households' presumably optimal savings decisions and the optimal capital stock. What about current account?

$$CA_t = B_{f,t+1} - B_{f,t} = S_t - I_t$$

A current account deficit arises when investment needs exceed aggregate savings.

Dynastic models VII

The evolution of B depends on consumption choices. Assume that the representative household maximizes

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t) L_t,$$

where C_t is per capita consumption (not intensive form but per L_t).

Dynastic models VII

The evolution of B depends on consumption choices. Assume that the representative household maximizes

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(C_t) L_t,$$

where C_t is per capita consumption (not intensive form but per L_t). The budget constraint every period is:

$$B_{t+1} + C_t L_t = (1 + r^*) B_t + w A_t L_t$$

where B_t is assets owned by the consumer at the end of period t.

Dynastic models VIII

The intertemporal budget constraint is

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r^*}\right)^t C_t L_t \leq (1+r^*)B_0 + \sum_{t=0}^{\infty}\left(\frac{1}{1+r^*}\right)^t W_t L_t$$

where I have imposed the no Ponzi-game condition.

Dynastic models VIII

The intertemporal budget constraint is

$$\sum_{t=0}^{\infty}\left(\frac{1}{1+r^*}\right)^t C_t L_t \leq (1+r^*)B_0 + \sum_{t=0}^{\infty}\left(\frac{1}{1+r^*}\right)^t W_t L_t$$

where I have imposed the no Ponzi-game condition. We notice that the present value of income can be written as

$$\sum_{t=0}^{\infty}\left(\frac{1+\gamma}{1+r^*}\right)^t wA_0 L_0,$$

so we need to assume $r^* > \gamma$ to avoid that the present value of labor income is infinite.

Dynastic models IX

OK. We find the optimal consumption path by maximizing utility subject to the budget constraint. First-order condition:

$$u'(C_t) = \beta(1 + r^*)u'(C_{t+1})$$

Dynastic models IX

OK. We find the optimal consumption path by maximizing utility subject to the budget constraint. First-order condition:

$$u'(C_t) = \beta(1 + r^*)u'(C_{t+1})$$

Then assume that $u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$. We write first-order condition as:

$$(c_t A_t)^{-\theta} = \beta(1 + r^*)(c_{t+1} A_{t+1})^{-\theta}$$

Dynastic models IX

OK. We find the optimal consumption path by maximizing utility subject to the budget constraint. First-order condition:

$$u'(C_t) = \beta(1 + r^*)u'(C_{t+1})$$

Then assume that $u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$. We write first-order condition as:

$$(c_t A_t)^{-\theta} = \beta(1 + r^*)(c_{t+1} A_{t+1})^{-\theta}$$

or simply

$$c_{t+1} = c_t \frac{[\beta(1 + r^*)]^{1/\theta}}{1 + g}$$

Dynastic models IX

OK. We find the optimal consumption path by maximizing utility subject to the budget constraint. First-order condition:

$$u'(C_t) = \beta(1 + r^*)u'(C_{t+1})$$

Then assume that $u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}$. We write first-order condition as:

$$(c_t A_t)^{-\theta} = \beta(1 + r^*)(c_{t+1} A_{t+1})^{-\theta}$$

or simply

$$c_{t+1} = c_t \frac{[\beta(1 + r^*)]^{1/\theta}}{1 + g}$$

Combining this with the budget constraint (now in intensive form);

$$(1 + \gamma)b_{t+1} + c_t = (1 + r^*)b_t + w,$$

we can solve for the optimal consumption path, and thus also the evolution of assets.

When is there a BGP?

We can finally ask: When does a balanced growth path exist for this economy? For the closed economy it was sufficient to impose homothetic preferences (as we have done here). A BGP requires all variables in intensive form to converge at constant steady state values.

When is there a BGP?

We can finally ask: When does a balanced growth path exist for this economy? For the closed economy it was sufficient to impose homothetic preferences (as we have done here). A BGP requires all variables in intensive form to converge at constant steady state values.

- From the budget constraint we see that constant b only works if c is constant

When is there a BGP?

We can finally ask: When does a balanced growth path exist for this economy? For the closed economy it was sufficient to impose homothetic preferences (as we have done here). A BGP requires all variables in intensive form to converge at constant steady state values.

- From the budget constraint we see that constant b only works if c is constant
- When is consumption per efficient worker constant? From the Euler equation we see that this only holds when

$$1 = \frac{[\beta(1 + r^*)]^{1/\theta}}{1 + g}$$

When is there a BGP?

We can finally ask: When does a balanced growth path exist for this economy? For the closed economy it was sufficient to impose homothetic preferences (as we have done here). A BGP requires all variables in intensive form to converge at constant steady state values.

- From the budget constraint we see that constant b only works if c is constant
- When is consumption per efficient worker constant? From the Euler equation we see that this only holds when

$$1 = \frac{[\beta(1+r^*)]^{1/\theta}}{1+g}$$

or

$$r^* = (1+\rho)(1+g)^{\theta} - 1 = r_c$$

where $\beta = 1/(1+\rho)$ is the discount rate.

The BGP case

Let us therefore assume $r^* = r_c$. What happens?

The BGP case

Let us therefore assume $r^* = r_c$. What happens?

- Net assets and net foreign assets are constant. Consumption per efficient worker is also constant and equal to

$$c = w + (r - \gamma)b$$

  (Can only spend some of the return on assets in order to keep B/AL constant)

The BGP case

Let us therefore assume $r^* = r_c$. What happens?

- Net assets and net foreign assets are constant. Consumption per efficient worker is also constant and equal to

$$c = w + (r - \gamma)b$$

  (Can only spend some of the return on assets in order to keep B/AL constant)

- If a country starts out with $b_f > 0$, it will keep assets positive forever. Hence it also runs a current account surplus forever:

$$ca = \gamma b_f$$

  which is necessary to keep net assets b from declining as time passes.

The BGP case

Let us therefore assume $r^* = r_c$. What happens?

- Net assets and net foreign assets are constant. Consumption per efficient worker is also constant and equal to

$$c = w + (r - \gamma)b$$

  (Can only spend some of the return on assets in order to keep B/AL constant)

- If a country starts out with $b_f > 0$, it will keep assets positive forever. Hence it also runs a current account surplus forever:

$$ca = \gamma b_f$$

  which is necessary to keep net assets b from declining as time passes.

Is the last point weird? Remember that it also runs a trade deficit.

The BGP case II

So restricting the world interest rate to r_c gives us a BGP. But this is not very satisfactory.

The BGP case II

So restricting the world interest rate to r_c gives us a BGP. But this is not very satisfactory.

- This makes the open economy steady state capital intensity the same as in the closed economy

The BGP case II

So restricting the world interest rate to r_c gives us a BGP. But this is not very satisfactory.

- This makes the open economy steady state capital intensity the same as in the closed economy
- Steady state consumption will differ, but only because of initial wealth or initial debt. $B_0 = 0$ makes steady state consumption the same as well (but the transition to steady state will differ)

The BGP case II

So restricting the world interest rate to r_c gives us a BGP. But this is not very satisfactory.

- This makes the open economy steady state capital intensity the same as in the closed economy
- Steady state consumption will differ, but only because of initial wealth or initial debt. $B_0 = 0$ makes steady state consumption the same as well (but the transition to steady state will differ)
- Can be argued that $r^* = r_c$ is reasonable if the rest of the world has the same preferences and same technology. But then it would be better to model the rest of the world?

What if $r^* \neq r_c$?

If $r^* \neq r_c$, then a BGP does not exist.

What if $r^* \neq r_c$?

If $r^* \neq r_c$, then a BGP does not exist.

- Assume $r^* < r_c$. For such a low interest rate the representative household chooses a declining consumption path. Consumption will tend to zero, and in the limit all income is spent on servicing the debt.

What if $r^* \neq r_c$?

If $r^* \neq r_c$, then a BGP does not exist.

- Assume $r^* < r_c$. For such a low interest rate the representative household chooses a declining consumption path. Consumption will tend to zero, and in the limit all income is spent on servicing the debt.
- Assume $r^* > r_c$. Now the country wants an increasing consumption path. By accumulating more and more assets, the country recieves higher and higher interest rate income. Assets tend to infinity.

What if $r^* \neq r_c$?

If $r^* \neq r_c$, then a BGP does not exist.

- Assume $r^* < r_c$. For such a low interest rate the representative household chooses a declining consumption path. Consumption will tend to zero, and in the limit all income is spent on servicing the debt.
- Assume $r^* > r_c$. Now the country wants an increasing consumption path. By accumulating more and more assets, the country recieves higher and higher interest rate income. Assets tend to infinity.
- Neither implication seems realistic.

What if $r^* \neq r_c$?

If $r^* \neq r_c$, then a BGP does not exist.

- Assume $r^* < r_c$. For such a low interest rate the representative household chooses a declining consumption path. Consumption will tend to zero, and in the limit all income is spent on servicing the debt.
- Assume $r^* > r_c$. Now the country wants an increasing consumption path. By accumulating more and more assets, the country recieves higher and higher interest rate income. Assets tend to infinity.
- Neither implication seems realistic.
- Schmitt-Grohe and Uribe (2003): Closing small open economy models, Journal of International Economics, 61, 163–185.

What if $r^* \neq r_c$?

If $r^* \neq r_c$, then a BGP does not exist.

- Assume $r^* < r_c$. For such a low interest rate the representative household chooses a declining consumption path. Consumption will tend to zero, and in the limit all income is spent on servicing the debt.
- Assume $r^* > r_c$. Now the country wants an increasing consumption path. By accumulating more and more assets, the country recieves higher and higher interest rate income. Assets tend to infinity.
- Neither implication seems realistic.
- Schmitt-Grohe and Uribe (2003): Closing small open economy models, Journal of International Economics, 61, 163–185.
- Better if we endogenize the interest rate? Standard result: The country with the smallest discount rate will accumulate assets while the other takes on debt. In the limit all wealth is owned by the patient country.

Outline

OLG setup

So the dynastic representative agent model has the unfortunate property that it only converges to a BGP in a knife-edge case. What about OLG models?

OLG setup

So the dynastic representative agent model has the unfortunate property that it only converges to a BGP in a knife-edge case. What about OLG models?

- Assume the same production function as before

OLG setup

So the dynastic representative agent model has the unfortunate property that it only converges to a BGP in a knife-edge case. What about OLG models?

- Assume the same production function as before
- Small open economy: r^* is exogenous

OLG setup

So the dynastic representative agent model has the unfortunate property that it only converges to a BGP in a knife-edge case. What about OLG models?

- Assume the same production function as before
- Small open economy: r^* is exogenous
- Hence the capital intensity and real wage per efficient worker follow from r^*

OLG setup II

We assume that each generation lives for two periods. A young person at time t maximizes utility

$$U = u(C_{y,t}) + \beta u(C_{o,t+1})$$

subject to a budget constraint

$$C_{y,t} + \frac{C_{o,t+1}}{1+r^*} = wA_t$$

(only work when young).

OLG setup II

We assume that each generation lives for two periods. A young person at time t maximizes utility

$$U = u(C_{y,t}) + \beta u(C_{o,t+1})$$

subject to a budget constraint

$$C_{y,t} + \frac{C_{o,t+1}}{1+r^*} = wA_t$$

(only work when young). First-order condition:

$$u'(C_{y,t}) = \beta(1+r^*)u'(C_{o,t+1})$$

OLG setup III

Assume the same utility function as before but let $\theta \to 1$ (=> log utility). The first-order condition can then be written as

$$C_{o,t+1} = \beta(1 + r^*)C_{y,t}$$

OLG setup III

Assume the same utility function as before but let $\theta \to 1$ (=> log utility). The first-order condition can then be written as

$$C_{o,t+1} = \beta(1 + r^*)C_{y,t}$$

Combine with budget constraint to find:

$$C_{y,t} = \frac{1}{1+\beta} wA_t$$
$$C_{o,t+1} = \frac{\beta}{1+\beta}(1 + r^*)wA_t$$

OLG setup III

Assume the same utility function as before but let $\theta \to 1$ (=> log utility). The first-order condition can then be written as

$$C_{o,t+1} = \beta(1 + r^*)C_{y,t}$$

Combine with budget constraint to find:

$$C_{y,t} = \frac{1}{1+\beta} wA_t$$
$$C_{o,t+1} = \frac{\beta}{1+\beta}(1 + r^*)wA_t$$

while savings of young and old are:

$$S_{y,t} = wA_t - C_{y,t} = \frac{\beta}{1+\beta} wA_t = \sigma wA_t$$
$$S_{o,t} = -S_{y,t-1}$$

where σ is the savings rate.

OLG setup IV

What are we interested in now? We want to know how net assets and net foreign assets evolve. Net assets must equal savings of the young:

$$B_{t+1} = L_t S_{y,t} = \sigma w A_t L_t \Rightarrow (1+\gamma)b = \sigma w$$

OLG setup IV

What are we interested in now? We want to know how net assets and net foreign assets evolve. Net assets must equal savings of the young:

$$B_{t+1} = L_t S_{y,t} = \sigma w A_t L_t \Rightarrow (1+\gamma)b = \sigma w$$

Net foreign assets continue to be the difference between total assets and capital:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

OLG setup IV

What are we interested in now? We want to know how net assets and net foreign assets evolve. Net assets must equal savings of the young:

$$B_{t+1} = L_t S_{y,t} = \sigma w A_t L_t \Rightarrow (1+\gamma)b = \sigma w$$

Net foreign assets continue to be the difference between total assets and capital:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

As in the dynastic model: Capital is determined independently of savings decisions. The country will have net foreign debt if the optimal capital stock is larger than the supply of savings from domestic households.

OLG setup V

Is there a BGP?

- We see that b, b_f and c are all constant no matter what value r^* takes.

OLG setup V

Is there a BGP?

- We see that b, b_f and c are all constant no matter what value r^* takes.
- While the dynastic model features a BGP only when $r^* = r_c$, the individual/OLG model has a BGP for any value of r^*!

OLG setup V

Is there a BGP?

- We see that b, b_f and c are all constant no matter what value r^* takes.
- While the dynastic model features a BGP only when $r^* = r_c$, the individual/OLG model has a BGP for any value of r^*!
- We go straight to the steady state after one period. If $b_f > 0$, the country runs a CA surplus forever (and a deficit forever if $b_f < 0$.

OLG setup V

Is there a BGP?

- We see that b, b_f and c are all constant no matter what value r^* takes.
- While the dynastic model features a BGP only when $r^* = r_c$, the individual/OLG model has a BGP for any value of r^*!
- We go straight to the steady state after one period. If $b_f > 0$, the country runs a CA surplus forever (and a deficit forever if $b_f < 0$.
- Difference from dynastic: No chance for b running off to infinity since agents have finite lives and no bequests.

OLG setup VI

Determinants of net foreign assets:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

OLG setup VI

Determinants of net foreign assets:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

- A larger savings rate or higher steady state real wage (i.e. income) will make b_f larger

OLG setup VI

Determinants of net foreign assets:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

- A larger savings rate or higher steady state real wage (i.e. income) will make b_f larger
- A higher capital intensity makes b_f more negative

OLG setup VI

Determinants of net foreign assets:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

- A larger savings rate or higher steady state real wage (i.e. income) will make b_f larger
- A higher capital intensity makes b_f more negative
- What about the effect of γ? A high natural growth rate makes it more likely that b_f is negative because the current generation saves too little per efficient worker of tomorrow.

OLG setup VI

Determinants of net foreign assets:

$$b_f = \frac{\sigma w}{1+\gamma} - k$$

- A larger savings rate or higher steady state real wage (i.e. income) will make b_f larger
- A higher capital intensity makes b_f more negative
- What about the effect of γ? A high natural growth rate makes it more likely that b_f is negative because the current generation saves too little per efficient worker of tomorrow.
- (But note also that the *savings-GDP ratio* it increasing in γ, so in OLG models fast-growing countries save more)

Current account and trade balance

The current account equals the difference between saving and investment, or equivalently the change in net foreign assets. We find:

$$ca = \gamma b_f$$

What about the trade balance? We use another definition of the current account:

$$CA_t = TB_t + r^* B_{f,t}$$

or, in intensive form:

$$tb = (\gamma - r^*) b_f$$

Current account and trade balance II

- For $r^* = \gamma$ we get $tb = 0$.

Current account and trade balance II

- For $r^* = \gamma$ we get $tb = 0$.
- If $r^* > \gamma$, then debtor countries ($b_f < 0$) run a trade surplus while creditors run a deficit (as expected).

Current account and trade balance II

- For $r^* = \gamma$ we get $tb = 0$.
- If $r^* > \gamma$, then debtor countries ($b_f < 0$) run a trade surplus while creditors run a deficit (as expected).
- But for $r^* < \gamma$, a debtor country will run a trade deficit forever: Free lunch!

Current account and trade balance II

- For $r^* = \gamma$ we get $tb = 0$.
- If $r^* > \gamma$, then debtor countries ($b_f < 0$) run a trade surplus while creditors run a deficit (as expected).
- But for $r^* < \gamma$, a debtor country will run a trade deficit forever: Free lunch!
- While a creditor country runs a surplus: Forced lunch?

Current account and trade balance II

- For $r^* = \gamma$ we get $tb = 0$.
- If $r^* > \gamma$, then debtor countries ($b_f < 0$) run a trade surplus while creditors run a deficit (as expected).
- But for $r^* < \gamma$, a debtor country will run a trade deficit forever: Free lunch!
- While a creditor country runs a surplus: Forced lunch?
- Two last points are linked to dynamic inefficiency, which we return to in a minute.

Outline

Government debt

Notation:

- Each worker faces a tax $\tau_t A_t$

Government debt

Notation:

- Each worker faces a tax $\tau_t A_t$
- The government spends $c_{g,t} A_t L_t$ every period

Government debt

Notation:

- Each worker faces a tax $\tau_t A_t$
- The government spends $c_{g,t} A_t L_t$ every period
- Government debt is denoted $B_{g,t}$ and the new market clearing condition for assets is $b_t = b_{f,t} + b_{g,t} + k$. Refer to $b_{g,t}$ as the debt ratio. So any increase in b_g (keeping b unaffected) will automatically lower net foreign assets.

Government debt II

- Law of motion for debt ratio:

$$(1+\gamma)b_{g,t+1} = (1+r^*)b_{g,t} + c_{g,t} - \tau_t$$

Government debt II

- Law of motion for debt ratio:

$$(1+\gamma)b_{g,t+1} = (1+r^*)b_{g,t} + c_{g,t} - \tau_t$$

- If we use the law of motion to solve for a constant debt ratio we find:

$$b_g = \frac{c_g - \tau}{\gamma - r}$$

Government debt II

- Law of motion for debt ratio:

$$(1+\gamma)b_{g,t+1} = (1+r^*)b_{g,t} + c_{g,t} - \tau_t$$

- If we use the law of motion to solve for a constant debt ratio we find:

$$b_g = \frac{c_g - \tau}{\gamma - r}$$

- For $r^* > \gamma$ this equation gives the necessary primary surplus to keep a constant debt ratio. A smaller constant surplus makes the debt ratio explode.

Government debt II

- Law of motion for debt ratio:

$$(1+\gamma)b_{g,t+1} = (1+r^*)b_{g,t} + c_{g,t} - \tau_t$$

- If we use the law of motion to solve for a constant debt ratio we find:

$$b_g = \frac{c_g - \tau}{\gamma - r}$$

- For $r^* > \gamma$ this equation gives the necessary primary surplus to keep a constant debt ratio. A smaller constant surplus makes the debt ratio explode.
- But if $r^* < \gamma$, then this condition shows what debt ratio we converge to *independent* of the primary surplus/deficit. The government may choose $c_g - \tau$ to be as large as possible (but constant), and the debt ratio will still converge since the natural growth rate exceeds the interest rate.

Government debt III

Surplus needed to keep a constant debt ratio. Gives us an idea about how large debt ratios that can be sustainable (given what is regarded as a realistic surplus)

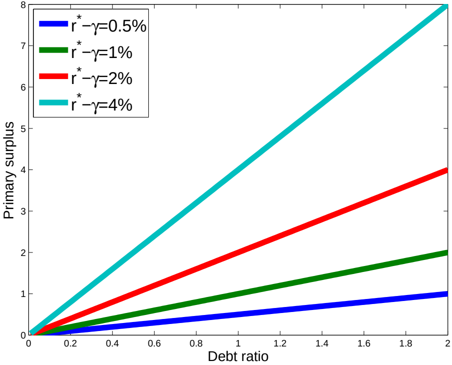


Figure: Surplus needed to keep a constant debt ratio when $r^* > \gamma$.

Government debt IV

What happens to debt when $r^* < \gamma$? Gives us an idea about how large debt burdens deficit countries will converge to. Vulnerable to higher interest rates/lower growth!

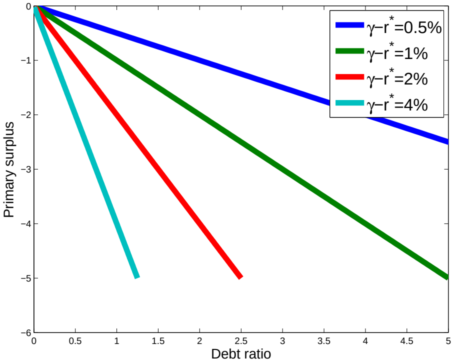


Figure: Steady state debt ratio as a function of surplus when $r^* < \gamma$.

Government debt V

What is *really* happening when $r^* < \gamma$?

- The government is always borrowing more: It borrows to pay the interest rate as well as to finance the deficit
- It is running a Ponzi-scheme – but it is somewhat more reasonable to allow Ponzi schemes from the government than from individuals. In fact, for $\gamma > r^*$ the only way to satisfy the NPG condition is by accumulating non-negative assets, which seems too strict.
- But unreasonable that the government can have any level of b_g without any effect on the interest rate.
- What about risk of default and the effect on r^*? Next class.

Outline

Pension system

To understand better the role of government debt, let us relate it to the presence of a pension system. Several differences between pension systems around the globe. Two most important distinctions:

- Funded vs. pay-as-you-go
- Defined contributions vs. defined benefits

Pension system: PAYG

Let q be the pension benefit. Assume that this is financed by the young paying a fixed tax rate τ. The benefit will be:

$$q = \frac{\tau w A_t L_t}{A_{t-1} L_{t-1}} = (1+\gamma)\tau w$$

Pension system: PAYG

Let q be the pension benefit. Assume that this is financed by the young paying a fixed tax rate τ. The benefit will be:

$$q = \frac{\tau w A_t L_t}{A_{t-1} L_{t-1}} = (1+\gamma)\tau w$$

The new budget constraint for a young person in period t is:

$$\begin{aligned} C_{y,t} + \frac{C_{o,t+1}}{1+r^*} &= (1-\tau) w A_t + \frac{1+\gamma}{1+r^*} \tau w A_t \\ &= \left(1 + \tau \left[\frac{(1+\gamma}{1+r^*} - 1\right]\right) w A_t \end{aligned}$$

Pension system: PAYG II

Aha!

Pension system: PAYG II

Aha!

- The old in the period the system is initiated will gain in any case.

Pension system: PAYG II

Aha!

- The old in the period the system is initiated will gain in any case.
- Lifetime income of the young will increase with τ if $\gamma > r^*$ (dynamic inefficiency)

Pension system: PAYG II

Aha!

- The old in the period the system is initiated will gain in any case.
- Lifetime income of the young will increase with τ if $\gamma > r^*$ (dynamic inefficiency)
- But if $r^* > \gamma$ then young plus all future generations have to pay for today's old to have a pension

Pension system: PAYG II

Aha!

- The old in the period the system is initiated will gain in any case.
- Lifetime income of the young will increase with τ if $\gamma > r^*$ (dynamic inefficiency)
- But if $r^* > \gamma$ then young plus all future generations have to pay for today's old to have a pension

Classic example of how a PAYG-system may raise welfare under dynamic inefficiency.

Pension system: PAYG III

In fact, if $\gamma > r^*$ then even the young will be willing to pay their entire wage income ($\tau = 1$). They are also willing to let the government issue an unlimited amount of debt in order to raise the contribution q to a level above $(1 + \gamma)\tau w$. Why? Same intuition for why the government can have an unlimited primary deficit (as long as it is stable): the situation with $r^* < \gamma$ permits the government to run a Ponzi scheme.

Pension system: PAYG IV

How will the pension system affect savings? Without the pension system consumption is:

$$C_{y,t} = \frac{1}{1+\beta} w A_t$$

Pension system: PAYG IV

How will the pension system affect savings? Without the pension system consumption is:

$$C_{y,t} = \frac{1}{1+\beta} wA_t$$

but with the pension system we have:

$$C_{y,t} = \frac{1}{1+\beta} \left(1 + \tau \left[\frac{(1+\gamma}{1+r^*} - 1\right]\right) wA_t$$

Pension system: PAYG IV

How will the pension system affect savings? Without the pension system consumption is:

$$C_{y,t} = \frac{1}{1+\beta} wA_t$$

but with the pension system we have:

$$C_{y,t} = \frac{1}{1+\beta} \left(1 + \tau \left[\frac{(1+\gamma}{1+r^*} - 1\right]\right) wA_t$$

so the saving of a young person relative to gross income is:

$$s = \frac{(1-\tau)wA_t - C_{y,t}}{wA_t} = (1-\tau)\frac{\beta}{1+\beta} - \tau \frac{1}{1+\beta} \frac{1+\gamma}{1+r^*}$$

Pension system: PAYG IV

How will the pension system affect savings? Without the pension system consumption is:

$$C_{y,t} = \frac{1}{1+\beta} wA_t$$

but with the pension system we have:

$$C_{y,t} = \frac{1}{1+\beta} \left(1 + \tau \left[\frac{(1+\gamma}{1+r^*} - 1\right]\right) wA_t$$

so the saving of a young person relative to gross income is:

$$s = \frac{(1-\tau)wA_t - C_{y,t}}{wA_t} = (1-\tau)\frac{\beta}{1+\beta} - \tau \frac{1}{1+\beta}\frac{1+\gamma}{1+r^*}$$

$\Rightarrow$ Savings do not just fall because of lower income when young (first term) but also because of higher income when old. Double effect!

Pension system: PAYG V

If the system is self-financed (i.e. no increase in government debt), then net foreign assets decline as much as young savings do. So countries with a PAYG-system should all else equal have less foreign assets/more debt than other countries.

Pension system: Funded system

What about a funded system? Setup would be:

- Young workers pay $\tau w A_t$.
- Old recieve $(1 + r)\tau w A_t$.

Effect on lifetime income? None at all! (This is similar (but not the same) to the Ricardian equivalence result).

Pension system: PAYG vs. funded

- So if $\gamma > r^*$ then a PAYG-system improves welfare while funded system has no impact

Pension system: PAYG vs. funded

- So if $\gamma > r^*$ then a PAYG-system improves welfare while funded system has no impact
- While if $\gamma < r^*$ then a PAYG-system improves welfare of the old in the first period while everyone else are worse off. Funded system still has no impact

Pension system: PAYG vs. funded

- So if $\gamma > r^*$ then a PAYG-system improves welfare while funded system has no impact
- While if $\gamma < r^*$ then a PAYG-system improves welfare of the old in the first period while everyone else are worse off. Funded system still has no impact
- For a funded system to be 'good', we need arguments that are outside the model, but for the young it is welfare-improving to move from PAYG to funded when $\gamma < r^*$.
- But will society gain from changing the system? Unless we decide to neglect the pension rights of today's old, then an extra tax must be levied on the young to finance the pension benefits of the old. Makes the two systems equivalent: Only difference is implicit vs. explicit debt.

Outline

Petroleum wealth

Our setup is fit for continuing Storesletten's discussion of how to manage petroleum wealth. Let $w_{p,t}$ measure the value of all future petroleum income, starting from the end of period $t-1$. It must be defined as:

$$w_{p,t} = \sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^{j+1} Y_{p,t+j}$$

where we ignore uncertainty (potentially important) and $Y_{p,t+j}$ measures petroleum revenues in period $t+j$.

Petroleum wealth II

Written in intensive form (note a small typo in Rødseth's note) we have:

$$w_{p,t} = \frac{1}{A_t L_t} \sum_{j=0}^{\infty} \frac{1}{1+r} Y_{p,t+j} = \frac{y_{p,t}}{1+r} + (1+\gamma)\frac{w_{p,t+1}}{1+r}$$

or

$$(1+\gamma)w_{p,t+1} = (1+r)w_{p,t} - y_{p,t}$$

So extraction reduces the value of future petroleum wealth. If $y_{p,t} = 0$ the wealth in intensive form increases if $r > \gamma$ but declines if $\gamma > r$.

Petroleum wealth III

What then about government *wealth* in total? We have:

$$w_{g,t} = w_{p,t} - b_{g,t}$$

Using the laws of motion for w_p and b_g:

$$\begin{aligned} (1+\gamma)w_{g,t+1} &= (1+r)w_{p,t} - y_{p,t} - (1+r^*)b_{g,t} - [c_{g,t} - \tau_t - y_{p,t}] \\ \Rightarrow (1+\gamma)w_{g,t+1} &= (1+r^*)w_{g,t} - [c_{g,t} - \tau_t] \end{aligned}$$

- Petroleum income should not be thought of as ordinary income since it is just book-keeping.
- But we should include the implicit interest rate income from the entire petroleum wealth in the equation

Petroleum wealth IV

Suppose we want to keep c_g and τ constant. This requires a constant w_g. Gives the following "spending rule":

$$c_g - \tau = (r^* - \gamma)w_g$$

Spend the part of the real return on government wealth that exceeds the natural growth rate.

Petroleum wealth V

But what is an optimal way to spend the oil wealth? Then we need a welfare criterion.

- Can think of the dynastic (repr. agent) model as a social planner model where the utilty function represents a welfare criterion (should be slightly modified to fit the OLG-setup here)
- What we have seen is that the social planner should follow the rule just described if it is on a BGP, i.e. if $r^* = r_c$, where

$$r_c = (1+\rho)(1+g)^{-\theta} - 1$$

  ρ now represents the social planner's discount rate.
- To have a rule that allows today's generation to spend more, we need $r^* < r_c$, and vice versa.
- Important that a proper model for an optimal spending rule takes into account the welfare loss associated with higher (and variable?) tax rates that may come in the future.

Petroleum wealth VI

Today's rule in Norway? *Handlingsregelen*: Spend the real return of the oil fund.

- Rule is applied to the fund, not total government wealth (i.e. not including the value of oil in the ground)
- Whole real return is spent, not adjusting for natural growth.

First point leads to an increasing spending path over time (until all oil has been pumped up). Second point will make the oil fund irrelevant relative to GDP in the very long run.

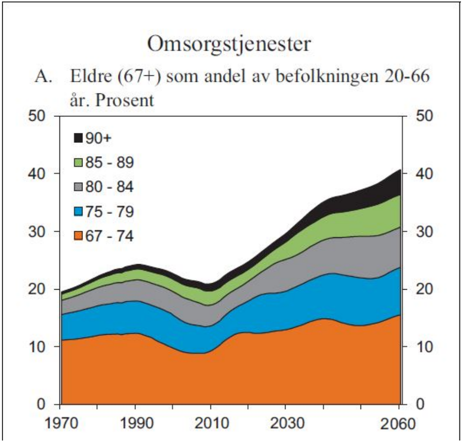
Outline

Perspektivmeldingen

Every 4th year, the Norwegian government publishes a report ("Perspektivmeldingen") with long run forecasts for economic growth and the development of public finances. Can we relate the main results of the report to our models from today?

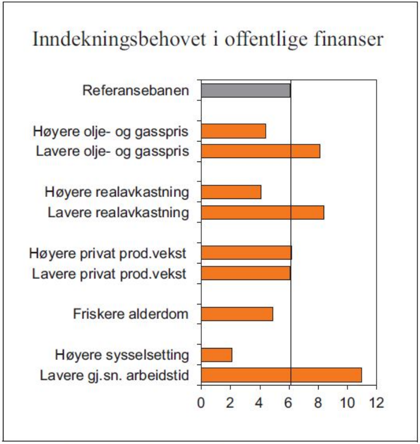
Perspektivmeldingen II

First of all, it is clear that something is happening to our natural growth rate (γ). Take the demographic development:



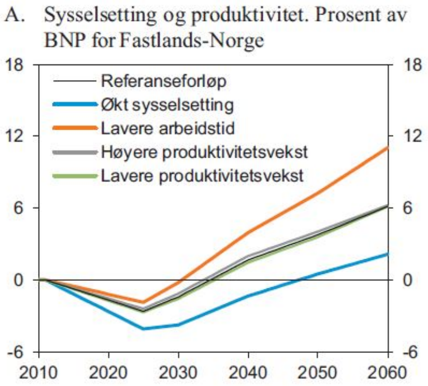
Perspektivmeldingen III

This has huge implications for the expected budget deficit as share of GDP in the future:

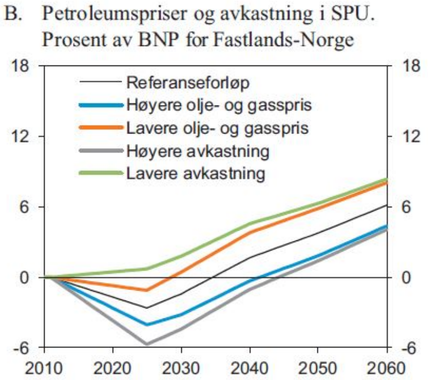


Figur 1.5 Behovet for inndekning i offentlige finanser i 2060 ved ulike utviklingsforløp. Prosent av BNP for Fastlands-Norge

Perspektivmeldingen IV

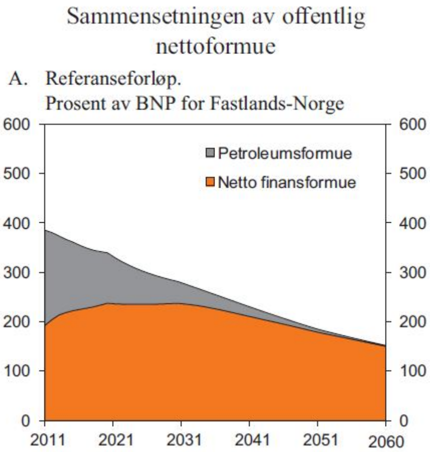


Perspektivmeldingen V



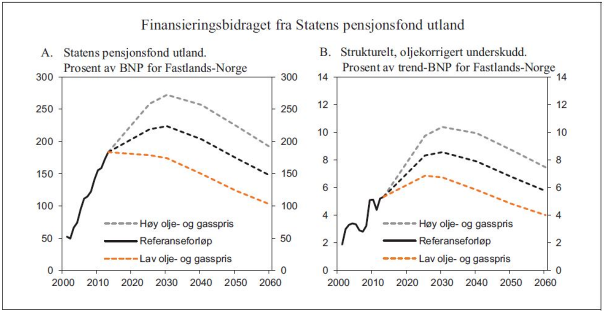
Perspektivmeldingen VI

The combined effect of *Handlingsregelen* and the depletion of oil reserves is clear from this figure:



Perspektivmeldingen VII

Oil revenues are important for government finances, but willbecome less so over time:



Figur 7.12 Finansieringsbidraget fra Statens pensjonsfond utland

Kilde: Finansdepartementet.

Perspektivmeldingen VIII

- PAYG-system always involves redistribution from the young to the old, but as long as $\gamma > r^*$ the young will be repaid when old
- But it is not clear that those who become old after 2060 will be repaid – hence it may be that the current system is a large redistribution from the generation born after 1980 to their parents' generation. Problem? γ has fallen! But real interest rates have fallen too so not clear that a funded system is better. But pension benefits must be adjusted to the drop in γ.
- We see that the expected deficit depends more on labor supply than productivity (contrary to our model where γ is what matters)

Perspektivmeldingen IX

- Without any change to the system, the deficit from around 2035 must be financed by spending more oil revenues than what *Handlingsregelen* allows for.
- Thus, even though the oil fund as share of GDP will decrease in any case, it may happen much faster
- Consistent with optimal policy? Depends on preferences of social planner.
- If politicians have a short perspective (4 years?), their discount rate is high. This raises r_c, making it more likely that $r_c > r^*$ so that they desire higher consumption today!