Government debt
Lecture 9, ECON 4310

Tord Krogh

September 10, 2013

Today's lecture

Topics:

- Basic concepts
- Tax smoothing
- Debt crisis
- Sovereign risk

Outline

Govt debt: Basic concepts

We start out with the government's budget constraint: In each period t government spending is g_t, taxes are τ_t, while debt issued is denoted d_{t+1}. The period t budget constraint is:

$$g_t = \tau_t + d_{t+1} - (1 + r_t)d_t$$

As derived in the seminar, we find the intertemporal budget constraint by re-writing the budget constraint:

$$d_t = \frac{1}{1 + r_t}\left[\tau_t - g_t + d_{t+1}\right]$$

and then replace d_{t+i} iteratively for $i = 1, 2, ..., T$. This gives:

$$d_t = \sum_{s=t}^{T} \frac{\tau_s - g_s}{\Pi_{i=t}^{s}(1 + r_i)} + \frac{d_{T+1}}{\Pi_{i=t}^{T}(1 + r_i)}$$

Govt debt: Basic concepts II

We impose the "no Ponzi" condition for the government:

$$\lim_{T\to\infty} \frac{d_{T+1}}{\Pi_{i=t}^{T}(1+r_i)} \leq 0$$

which requires the level of debt to grow slower than the interest rate in the long run. With this, the intertemporal budget constraint is:

$$d_t \leq \sum_{s=t}^{\infty} \frac{\tau_s - g_s}{\Pi_{i=t}^{s}(1+r_i)}$$

If we assume that the government never taxes more than necessary we can replace $\leq$ by $=$ in the above equations.[1]

[1] Romer uses continuous time notation, but otherwise everything is the same.

Govt debt: Basic concepts III

What does the no Ponzi condition mean? Assume a constant interest rate. Then

$$\Pi_{i=0}^{T}(1+r_i) = (1+r)^{T+1}$$

Further, assume that debt grows at a constant rate δ:

$$d_t = (1+\delta)^t d_0$$

Then we see that:

$$\frac{d_{T+1}}{\Pi_{i=0}^{T}(1+r_i)} = \left(\frac{1+\delta}{1+r}\right)^{T+1} d_0$$

Govt debt: Basic concepts IV

So the no Ponzi condition:

$$\lim_{T \to \infty} \left(\frac{1+\delta}{1+r} \right)^{T+1} d_0 \leq 0$$

is satisfied with equality for $\delta < r$.

Govt debt: Basic concepts V

Can we write models where the no Ponzi condition is violated in equilibrium? Return to the OLG models that we have seen before.

- Here the discounted value of future debt may converge to a positive level!
- Necessary condition: Dynamic inefficiency (when the real interest rate is lower than the growth rate of the economy).

Outline

Taxes and debt

Next we will consider a theory for what determines the deficit (and therefore the level of debt).
Recall: Under Ricardian equivalence, the timing of taxes is irrelevant. This also means that the deficit doesn't matter.

Tax smoothing

In the model we develop, Ricardian equivalence will fail because taxes are distortionary. Suddenly the deficit starts to matter. We will ask the question:

- What is the optimal path of taxes, $\{\tau_s\}_{s=t}^{\infty}$, for a given level of expenditure?

'Tax smoothing' comes out as the answer. Main point: For a given path of expenditure, the government should choose the most efficient tax scheme (which turns out to be a smooth tax rate).

Tax smoothing II

By pinning down the opitmal path of taxes, this also determines the deficit. The explanation for deficits/surpluses becomes:

- Surpluses are due to periods of high output, such that the optimal tax revenue exceeds expenditure
- Deficits are due to periods of low output, such that the optimal tax revenue is lower than expenditure

[This sounds very different from the political debate in Europe and US]

Tax smoothing III

OK, so in this model taxes are *distortionary*. For simplicity, assume that the (welfare) cost of taxes, denoted L_t, are given by:

$$L_t = f(\frac{\tau_t}{Y_t})Y_t$$

where τ_t is taxes and Y_t is output. We see that the cost relative to output (L_t/Y_t) is determined as a function of taxes relative to output (τ_t/Y_t). Assume that $f(\bullet)$ is convex (so the increase in welfare loss of higher taxes is larger when taxes are already large).

Tax smoothing IV

Let us follow Romer in looking at first the perfect foresight case, and then a simple version with uncertainty. Common assumptions:

- Constant interest rate r
- No ponzi condition for government holds
- Initial debt d_0 given
- Government chooses the path of taxes to minimize the (expected) discounted sum of welfare losses

Tax smoothing: Perfect foresight

Perfect foresight. Income for period t is an exogenous variable Y_t (known in advance). Path of expenditure $\{g_t\}_{t=0}^{\infty}$ is fixed. The intertemporal budget constraint of the government is

$$d_0 = \sum_{t=0}^{\infty} \frac{\tau_t - g_t}{(1+r)^{t+1}}$$

We therefore have the following optimization problem:

$$\min_{\{\tau_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} Y_t f(\frac{\tau_t}{Y_t})$$

$$s.t.$$

$$d_0 = \sum_{t=0}^{\infty} \frac{\tau_t - g_t}{(1+r)^{t+1}}$$

which has Lagrangian:

$$L = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} Y_t f(\frac{\tau_t}{Y_t}) - \lambda(\sum_{t=0}^{\infty} \frac{\tau_t - g_t}{(1+r)^{t+1}} - d_0)$$

Tax smoothing: Perfect foresight II

Differentiating with respect to τ_t yields:

$$\frac{1}{(1+r)^t} Y_t f'(\frac{\tau_t}{Y_t}) \frac{1}{Y_t} - \lambda \frac{1}{(1+r)^{t+1}} = 0$$

which reduces to

$$(1+r) f'(\frac{\tau_t}{Y_t}) = \lambda$$

This must hold for all t. Hence:

$$f'(\frac{\tau_0}{Y_0}) = f'(\frac{\tau_t}{Y_t})$$

for $t = 1, 2, 3, \ldots..$ When f is strictly increasing, this implies

$$\frac{\tau_0}{Y_0} = \frac{\tau_t}{Y_t}$$

Meaning? *Distortions are minimized when taxes relative to output is a constant ratio*. (Constant tax rate).

Uncertainty

What happens if there is uncertainty about future output? It is natural that the government now tries to minimize

$$E_0\left[\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}Y_t f(\frac{\tau_t}{Y_t})\right]$$

i.e. the expected welfare loss. Assume that the interest rate is still constant, so that only g_t and Y_t are uncertain. The constraint we maximize with respect to is then

$$d_0=\sum_{t=0}^{\infty}\frac{\tau_t-E_0[g_t]}{(1+r)^{t+1}}$$

[Allowing g_t to be uncertain means that it may depend on Y_t, but it doesn't change anything if it is purely exogenous.]

Uncertainty II

The first-order condition for τ_t now becomes

$$(1+r)E_0[f'(\frac{\tau_t}{Y_t})] = \lambda$$

which holds for all t. Combine the FOC for t with that for period 0:

$$f'(\frac{\tau_0}{Y_0}) = E_0[f'(\frac{\tau_t}{Y_t})]$$

Then assume, as Romer, that f is quadratic, making f' linear. Since $E[aX] = aE[X]$ when a is a constant, this gives:

$$\frac{\tau_0}{Y_0} = E_0[\frac{\tau_t}{Y_t}]$$

Meaning? *Expected value of distortions is minimized when taxes relative to output is expected to be a constant ratio.* (Constant tax rate).

Tax rate

The optimal tax rate, τ^*, is found by inserting for $\tau_t = \tau^* Y_t$ in the budget constraint:

$$d_0 = \sum_{t=0}^{\infty} \frac{\tau^* E_0[Y_t] - E_0[g_t]}{(1+r)^{t+1}}$$

and then solving for τ^*:

$$\tau^* = \frac{d_0 + \sum_{t=0}^{\infty} \frac{E_0[g_t]}{(1+r)^{t+1}}}{\sum_{t=0}^{\infty} \frac{E_0[Y_t]}{(1+r)^{t+1}}}$$

where $E_0[g_t] = g_t$ and $E_0[Y_t] = Y_t$ if there is perfect foresight.

- The optimal tax rate is equal to the ratio of NPV of debt and expenditure relative to NPV of output

Implications

What are the implications? It implies that taxes should be set to smooth the tax burden over time. Rather than adjusting taxes when expenditure is fluctuating, one should let *debt* play that role. This resembles 'automatic stabilizers', but motivation is *not* to stabilize business cycles. Here the motivation is to minimize welfare losses associated with taxation.

Implications II

- Since τ_t / Y_t is (expected) to be constant over time, we will see large deficits relative to GDP when g_t / Y_t is unusually large.
- Obvious source to such situations: Wars

Outline

Debt crisis

In the tax smoothing model, we looked at optimal timing of taxes for a given interest rate r and complete access to borrowing at all times. Not the case in practice. We therefore consider a simple model for debt crisis. We will have a government with:

- An amount of debt D coming due (it must be repaid)
- It plans to *roll over* the debt (i.e. issue new debt) for one period
- Then use tomorrow's tax revenue, T, to pay the debt
- But the tax revenue is stochastic. T has cdf $F(\bullet)$, implying that $P(T \leq t) = F(t)$.
- This makes it possible that the government must default

Debt crisis II

Assumptions:

- Investors are risk neutral
- There is a risk-free (gross) interest rate $\bar{R}$ (unspecified where that comes from!)
- Government offers a gross interest rate $R = 1 + r$
- If $T \geq RD$, the government repays
- If $T < RD$, the government defaults on the *entire* debt

Debt crisis: Equilibrium conditions

How do we analyze this model? First think of the link between R and the probability of default, denoted π.

- In equilibrium R will adjust such that the expected return on government debt equals the risk free rate
- Formally this means

$$(1 - \pi)R = \bar{R}$$

- Why not different? If $(1 - \pi)R > \bar{R}$, the government can save money by offering a lower rate. If $(1 - \pi)R < \bar{R}$, the investors simply go for the risk-free alternative.

This condition therefore gives R as a convex function of π. When $\pi = 0$, $R = \bar{R}$, but if $\pi \to 1$ we get $R \to \infty$.

Debt crisis: Equilibrium conditions II

Second think about the link between tax revenues and the probability of default.

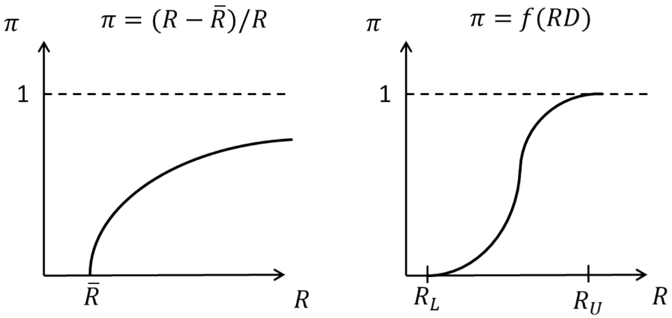
- Investors' perception of the probability of default must be based on $F(\bullet)$
- Since the government only defaults when taxes fall short of RD, we have:

$$\pi = F(RD)$$

For a symmetric distribution, this gives π as an S-shaped function of R. Let R_L denot the level for which $\pi = 0$, and R_U the level for which $\pi = 1$.

Debt crisis: Equilibrium conditions III

Draw each condition in graphs:



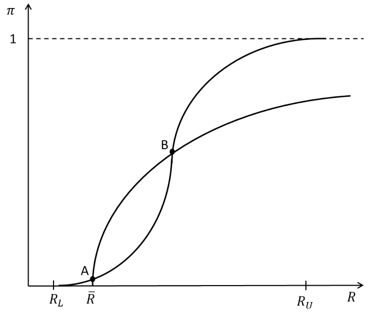
Debt crisis: Equilibrium

In equilibrium both conditions must be satisfied. Assume $R_L < \bar{R} < R_U$. Since both conditions are satisfied in equilibrium we know that:

- Investors get the risk free return in expectation
- And expectations are consistent with the cdf of tax revenues

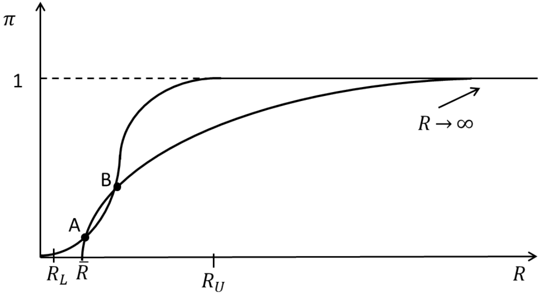
Easiest to look at it graphically.

Debt crisis: Multiple equilibria



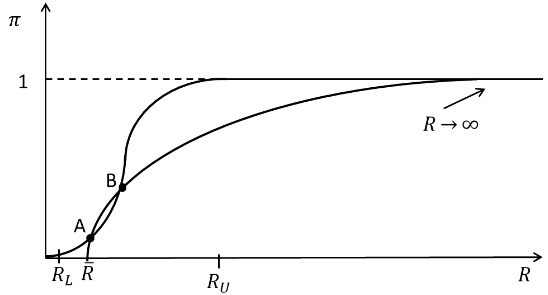
This model has multiple equilibria. In point A, we get an equilibrium where both the interest rate and probability of default is low. But in point B, the interest rate is much higher, and default is more likely.

Debt crisis: Multiple equilibria II



But it is also possible to have an equilibrium where $\pi = 1$ and $R \to \infty$! This is when the market 'shuts down': Investors are unwilling to buy government debt, no matter what interest rate they are offered. Further, their fear of default is justified by extremely high interest rate factors.

Debt crisis: Stability

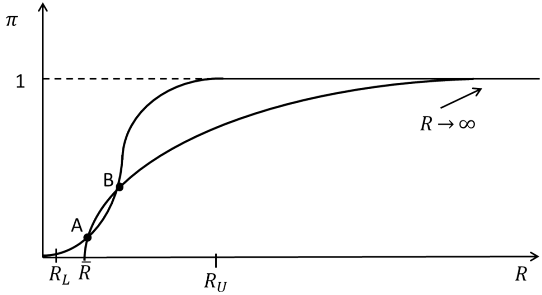


So we can think of it as being three possible equilibria:

- Normal times
- Distress
- Crisis

Are any of the equilibria stable?

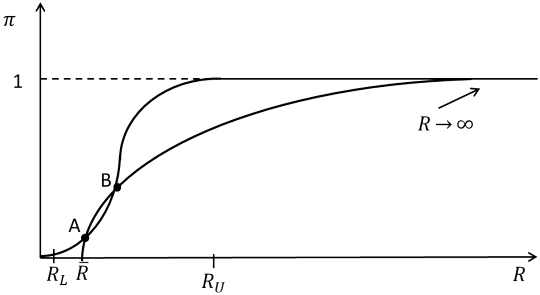
Debt crisis: Stability II



Consider point B. If investors suddenly perceive the probability to be slightly below π_B, what happens?

- They require a return R lower than R_B
- At this return, the probability of default is even lower than what they first thought
- So they will most likely require an even lower return
- This process gets us down to point A.

Debt crisis: Stability III



What if investors suddenly perceive the probability to be slightly *above* π_B?

- They require a return R higher than R_B
- At this return, the probability of default is *higher* than what they first thought
- So they will most likely require an even higher return
- This process pushes us towards the complete crisis equilibrium!

Debt crisis: Stability IV

- So B is an unstable equilibrium
- But normal times and crisis times are stable equilibria
- Can interpret point B as the ‘tipping point’.
- Fluctuations in R (or π) close to point A will not be harmful
- But sudden shifts might send you over to the crisis stage
- The shifts in π or R that lead to crisis can be unrelated to fundamentals (self-fulfilling prophesies)
- But ‘fundamentals’, such as a large value of D will also make default more likely
- Default is always ‘unexpected’ since there is no stable equilibrium with large value of π (except when there is default!)

Outline

Sovereign risk

To end the discussion of government debt, we look quickly at some more open economy issues.[2] *Sovereign risk* refers to the possibility of government default and seizure of foreign assets (in the country). Is a natural part of an inter-connected world economy since there is no institutional framework that exist to legally enforce countries to stand by their obligations.

[2]Reference for those who want to learn more: Obstfeld and Rogoff, 1996.

Sovereign risk II

Still some ways to enforce payments:

- Reject defaulting countries access to credit markets in the future/higher interest rates due to default risk
- Trade sanctions

For simplicity, let us assume that sanctions after a default involves confiscation of an η share of output and assets.

Sovereign risk III

If K_t is the capital stock and $F(K_t)$ is the production function, if the country defaults in period t the creditors will manage to get

$$\eta(F(K_t) + K_t)$$

back through different sanctions.

Two-period model with default

To illustrate some simple ideas, we write down a two-period representative agent model.

- Utility function is standard: $U = u(C_1) + \beta u(C_2)$
- The representative agent starts out with capital K_1, which produces $Y_1 = F(K_1)$
- It must decide how much to invest and consume
- If there is no default risk, it can borrow and lend internationally at the interest rate r

Two-period model with default II

So without default risk, the model is described by the following optimization problem:

$$\max_{C_1,K_2} u(C_1) + \beta u\left([1+r](F(K_1) - C_1) + F(K_2) + K_1 - r(K_2 - K_1)\right)$$

The first-order conditions to this problem are:

- The standard Euler equation: $u'(C_1) = \beta u'([1+r](F(K_1) - C_1) + F(K_2) + K_1 - r(K_2 - K_1))$
- and the optimal investment condition: $F'(K_2) = r$

Two-period model with default III

What happens when there is default risk?

- Let $-B_2$ denote the amount borrowed from abroad
- Without default, $-(1+r)B_2$ was always repaid
- Now, when period 2 arrives the agents will now only pay R:

$$R = \min\{-(1+r)B_2, \eta(F(K_2)+K_2)\}$$

Here we see that the country only repays the full loan with interest if it is less than the cost of not doing so. If η is very small, the country 'always' defaults.

Two-period model with default IV

Will discuss three issues in light of this model

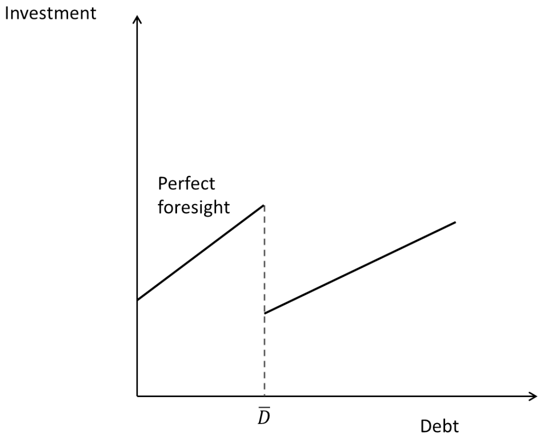
- Debt ceiling
- Debt overhang
- Debt Laffer curve

Debt ceiling

Result [not to be derived]: A country with default risk will face an endogenous *debt ceiling*. It will never get to borrow more than $\bar{D}$.

Debt ceiling II

What happens if the country gets to lend $\bar{D} + \Delta$? The optimal rate of investment will fall (a lot), since the country will default in any case, making it less attractive to have period 2 output and assets. The fall will be discontinuous.



Debt ceiling III

Intuition?

- If the country defaults for sure, the return from investment is only $(1-\eta)F'(K_2)$

Effect: A debt ceiling illustrates that presence of sovereign risk may limit a country's access to international borrowing. This will cause inefficiency if the debt ceiling is binding, since then the country is unable to invest the optimal amount.

Debt overhang

The second issue we'll discuss is the effect of starting out with a huge debt burden, and how sovereign risk will then impede growth. In our two-period model, assume therefore that $-B_1 = D > 0$, so the country starts out with a given level of debt. Let the utility function be (the very simple)

$$U = C_1 + E(C_2)$$

and take period 1 output as given, while period 2 output is $A_2 F(K_2)$, where $K_2 = I_1$ (capital depreciates completely after one period) and A_2 is random. Further, assume that the world interest rate is zero ($r = 0$).

Debt overhang II

The period-by-period budget constraints facing the country are:

$$C_1 + K_2 = Y_1$$
$$C_2 = A_2F(K_2) - \min[\eta A_2F(K_2), D]$$

(Since utility is linear, it will never bother to borrow any extra from abroad)
Inserting for these conditions, the country will choose K_2 in order to maximize:

$$Y_1 - K_2 + \mathbf{E_t}\{A_2F(K_2) - \min[\eta A_2F(K_2), D]\}$$

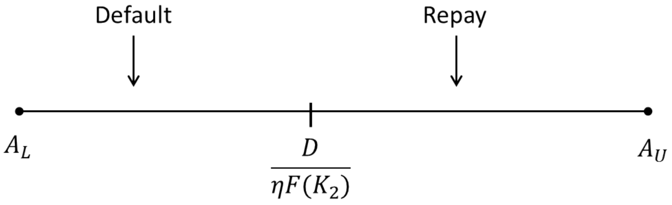
Debt overhang III

Assume that A_2 has distribution $\pi(A_2)$ over $A_2 \in [A_L, A_U]$ with $\mathbf{E_t}(A_2) = 1$. This makes $\mathbf{E_t}\left\{A_2 F(K_2)\right\} = F(K_2)$, such that the maximization problem is simply:

$$\max_{K_2} Y_1 - K_2 + F(K_2) - \mathbf{E_t}\left\{\min[\eta A_2 F(K_2), D]\right\}$$

Debt overhang IV

What is this expected value? For a given level of K_2, we understand that whether the country defaults or repays depends on A_2:



Debt overhang V

When it defaults, the creditors get $\eta A_2 F(K_2)$. If it repays, they get D. The expected value is therefore given by the function $V(D, K_2)$:

$$\mathbf{E_t}\left\{\min[\eta A_2 F(K_2), D]\right\} = V(D, K_2) = \eta F(K_2)\int_{A_L}^{\frac{D}{\eta F(K_2)}} A_2 \pi(A_2) dA_2 + D\int_{\frac{D}{\eta F(K_2)}}^{A_U} \pi(A_2) dA_2$$

Interpretation? If productivity is high enough, debt is repaid and everything is fine. But if productivity is low, the country ends up defaulting. In those cases a share η of output is 'taxed' by foreign creditors.

Debt overhang VI

The effect of debt overhang can be seen from the first-order condition for K_2:

$$F'(K_2)\left[1 - \eta \int_{A_L}^{\frac{D}{\eta F(K_2)}} A_2 \pi(A_2) dA_2\right] = 1$$

(for derivation see p. 393 and footnote 43 in Obstfeld and Rogoff, 1996).
The possibility of default makes the country invest less than the optimal amount (which would give $F'(K_2) = 1$). This is because what the creditors get is proportional to output when there's default.

Debt overhang VII

This shows how a large initial stock of debt depresses investment activity. Possibility of default creates an uncertain investment environment.

Debt Laffer curve

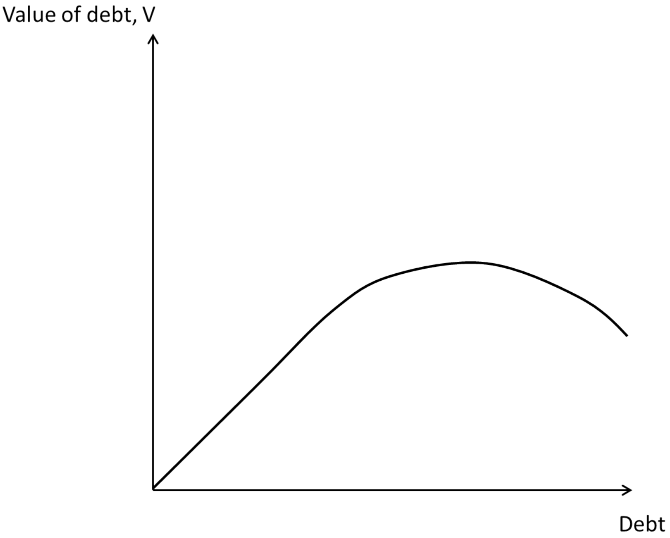
Realizing that countries may suffer from a debt overhang effect; what is it optimal for creditors to do? Consider the creditors of Greece. If they cut the debt by Δ:

- They have a direct loss of Δ if the loan is repaid
- But this may reduce the overhang effect, and make default less likely

The last effect can dominate!

Debt Laffer curve II

Implies a debt Laffer curve, as discussed by Krugman (1989) and Sachs (1989).



Debt Laffer curve III

Challenges:

- How to coordinate the debt writedown?
- No proper coordinator on the international level
- Even in Europe: very difficult