(Incomplete) summary of the course so far

Lecture 9a, ECON 4310

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Main topics

This semester we will go through:

- Ramsey (check)
- OLG (check)
- RBC methodology for analyzing business cycles (later)
- Search and matching models (later)
- Applications of microfounded intertemporal models to discuss e.g.
    - Optimal taxation (intertemporally – not the optimal combination of taxes) (today)
    - Debt crisis (today)
    - Tobin's q theory (future)
    - Savings, risk and asset prices (future)

Intention?

1. Give you the necessary background to understand 'modern' macroeconomic theories – which are all developed using the same microfounded intertemporal approach as the Ramsey and OLG models.
2. Make you capable of solving RBC models for the business cycle – involving derivation of first-order conditions and lineariziation – which you can use to understand the sources of economic fluctuations.
3. Understand important implications one can derive with regards to asset pricing and optimal taxation principles
4. Know central theories for explaining unemployment

For most of you, this course is quite challenging, since the models are extremely different from the tradititional 'Keynesian' models that you learn at the bachelor level.

Outline

Ramsey model

The basic Ramsey model is also known as the *neoclassical growth model*.

- Think of it as an extended Solow model. We no longer assume a constant savings rate. Instead we microfound the savings-decision
- Important ingredient: Agents are assumed to have *rational expectations*
- If there is no uncertainty this implies *perfect foresight*

Ramsey model II

Solution method: Solve the social planner's problem

- Second Welfare Theorem: A social planner can achieve any Pareto optimal allocation
- With several agents, the wanted allocation is obtained by choosing the appropriate *welfare weights*
- With a representative agent, the Pareto optimum is unique

Ramsey model III

Consider the deterministic case:

$$\max_{\{c_s,k_{s+1}\}_{s=t}^{\infty}} \left\{ \sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \right\}$$

$$s.t.$$

$$c_t + k_{t+1} = Ak_t^{\alpha} n_t^{1-\alpha} + (1-\delta)k_t$$

$$c_t \geq 0$$

$$k_{t+1} \geq 0$$

$$0 \leq n_t \leq 1$$

with $k_t > 0$ given. Can simplify by setting $n_t = 1$ and ignore $c_t \geq 0$ and $k_{t+1} \geq 0$ under 'normal' assumptions.

Ramsey model III

Lagrangian is:

$$L = \sum_{s=t}^{\infty} \left[\beta^{s-t} u(c_s) - \lambda_s \left(c_s + k_{s+1} - Ak_s^{\alpha} + (1-\delta)k_s \right) \right]$$

and the first-order conditions with resepect to c_s and k_{s+1} are:

$$\begin{aligned} c_s : &\quad \beta^{s-t} u'(c_s) - \lambda_s = 0 \\ k_{s+1} : &\quad -\lambda_s + \lambda_{s+1} \left(1 - \delta + \alpha A k_{s+1}^{\alpha-1} \right) = 0 \end{aligned}$$

Ramsey model IV

Combine the first-order conditions to see that optimum is characterized by the Euler equation:

$$u'(c_t) = \beta[1 - \delta + \alpha A k_{t+1}^{\alpha-1}]u'(c_{t+1})$$

the resource constriant:

$$c_t + k_{t+1} = A k_t^{\alpha} + (1 - \delta)k_t$$

as well as a transversality condition:

$$\lim_{T \to \infty} \beta^T u'(c_T) = 0$$

Ramsey model V

In a steady state we have $c_t = c_s = c_*$ and $k_t = k_s = k_*$ (no technological or population growth). From the Euler equation we get

$$(*) \quad \alpha A k_*^{\alpha-1} = \rho + \delta$$

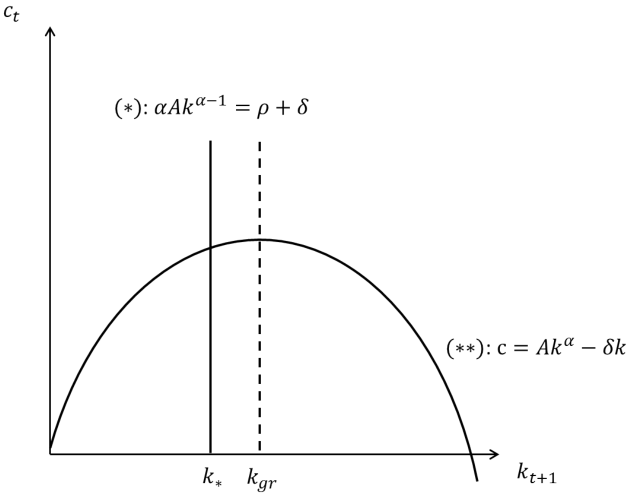
where $\rho = \frac{1}{\beta} - 1$. From the resource constraint we have

$$(**) \quad A k_*^{\alpha} - \delta k_* = c_*$$

Note that $k_* < k_{gr}$ (the golden-rule level of capital) as long as $\rho > 0$.

Ramsey model VI

We can draw $(*)$ and $(**)$ in a k, c diagram



Ramsey model VII

Saddle path: By showing which way consumption and capital moves when we are off the steady state curves, we trace out the saddle path. Can use the phase diagram to analyze the effects of an exogenous shift in e.g. productivity (A).

Ramsey model VIII

Extensions:

- Solve the model under labor and productivity growth
- Labor supply
- How to work with stochastic productivity

OLG

What about overlapping generations? Gives us models where

- The first welfare theorem no longer holds
- Ricardian equivalence is broken
- Good framework for studying rational bubbles

OLG II

How to solve such models?

- First solve the problem for each single agent
- Then see what it implies for the aggregate

OLG III

Important results:

- Dynamic inefficiency is possible

$\Rightarrow$ Government may improve welfare

- A bubble might also improve welfare
- As illustrated by open economy case: Perhaps a better model for studying the development of net asset positions for countries than a representative agent setup.

Outline

Taxation and debt

The Ramsey and OLG framework you have learned is useful to discuss a number of issues. Among them: What is the best policy for financing government expenditure using taxes or debt?

Taxation and debt II

1. In basic frictionless Ramsey model: Irrelevant question (Ricardian equivalence)
2. In OLG model: Not irrelevant – but answer depends on social planner's preference for intergenerational equity
3. With distortinary taxes: Tax smoothing optimal. Level of debt follows residually. Debt grows in periods it is optimal to have relatively low taxes
4. With default risk: Migth be optimal to limit the level of debt to reduce the likelihood of self-fullfilling crisis

You have been through the first two before. Today we are dealing with the two last.

Taxation and debt, Case 1

Under Ricardian equivalence, the timing of taxes and government debt is irrelevant. Implication: You might as well finance increases in government expenditure using taxes instead of issuing debt. $\Rightarrow$ If true, you should doubt the potential for expansionary fiscal policy.

Taxation and debt, Case 1: II

How to see that Ricardian equivalence holds?

- First look at the intertemporal BC of the government
- Then see how the agent makes optimal choices
- Derive the intertemp. BC of the agent

Taxation and debt, Case 1: III

Consider an *infinitely lived* government with expenditure g_t, tax revenue tax_t and government assets b_{t+1}. Period t budget constraint for a constant interest rate r:

$$g_t + b_{t+1} = tax_t + (1+r)b_t$$

Intertemporal budget constraint, starting in period 0 (assuming no Ponzi-game condition holds):

$$(1+r)b_0 = \sum_{t=0}^{\infty} \frac{g_t - tax_t}{(1+r)^t}$$

Assuming $b_0 = 0$, this implies

$$\sum_{t=0}^{\infty} \frac{g_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{tax_t}{(1+r)^t}$$

Taxation and debt, Case 1: IV

What about the agent? Assume that the agent maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_t\right]$$

subject to

$$c_t + a_{t+1} = w - tax_t + (1+r)a_t$$

Assume $\beta(1+r) = 1$. In the frictionless model **lump-sum taxes** are possible. So tax_t is taken as given by the agent. The first-order condition for optimum is:

$$c_t = c_{t+1}$$

This implies a constant consumption level ($\bar{c}$).

Taxation and debt, Case 1: IV

The intertemporal budget constraint (imposing no Ponzi-game), starting from period 0 is:

$$(1+r)a_0 = \sum_{t=0}^{\infty} \frac{c_t - w + tax_t}{(1+r)^t}$$

Then we can insert for optimal c_t and also use the government budget constraint to write

$$(1+r)a_0 = \sum_{t=0}^{\infty} \frac{\bar{c} - w + g_t}{(1+r)^t}$$

Taxation and debt, Case 1: V

Hence with lump-sum taxes

- The optimal choice of consumption is independent of how taxes are timed
- Could also be extended to labor supply
- Whether you choose to set $tax_t = g_t$ every period, or $tax_t = constant$ and let deficits vary with the business cycle, it doesn't matter.
- Why not? Since the level of a lump-sum tax will not affect the consumption trade-off in the Euler equation or the consumption-leisure trade-off in the intratemporal optimality condition (if we have labor supply).
- The presence of a government will still have an impact, since it reduces $\bar{c}$. But it is only the NPV (i.e. total costs) that matters
- **The economy features Ricardian equivalence**

Taxation and debt, Case 2

Then we turn to an OLG model. For simplicity, let us use 2 generations. Only work when young. A working agent earns w. Young will pay tax_t in taxes and recieve p_t if they are old. Consumption is c_t^y when young and c_{t+1}^o when old. Savings are a_{t+1}. Budget constraints:

$$c_t^y + a_{t+1} = w - tax_t$$
$$c_{t+1}^o = p_t + (1+r)a_{t+1}$$

or, if we combine the two:

$$c_t^y + \frac{c_{t+1}^o}{1+r} = w - tax_t + \frac{p_t}{1+r}$$

Taxation and debt, Case 2: II

The government is still infinitely lived. Hence the intertemporal BC it faces (assuming no assets initially) will be

$$\sum_{t=0}^{\infty} \frac{p_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{tax_t}{(1+r)^t}$$

Even though the NPV of taxes equals NPV of expenditure, this is of little 'comfort' for an agent that only lives for two periods (he might benefit from this, of course).

Taxation and debt, Case 2: III

Ricardian equivalence will now fail to hold:

- Even when lump-sum taxes are available, the government may transfer income from one generation to the other. $\Rightarrow$ Timing of taxes will have effects.
- If taxes are distortionary as well, this is an extra reason for why it fails

Outline

RBC methodology

Ambition: To start out with the simplest possible neoclassical growth model, calibrate it to match important long-run moments, and then see how well it does in accounting for business cycle facts. Basic steps:

- Write down the optimization problem
- Find first-order conditions
- Characterize steady state
- Calibrate the model
- Linearize optimality conditions around steady state
- Solve the set of linearized equations
- Plot impulse-response functions, simulate, calculate moments etc.

Unemployment

How to think about involuntary unemployment in microfounded equilibrium models? Completely missing from RBC models. Two prominent theories that can help us understand unemployment better:

- Efficiency wages (Shapiro-Stiglitz)
- Search and matching models