Exercises to Seminar 2
ECON 4310

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1 Life-cycle behavior

1. A consumer lives for two periods. He starts life without any financial assets and leaves no bequests. His (exogenous) labor income is W_1 in period 1, W_2 in period 2, while his consumption in the two periods is C_1 and C_2. His utility function is

$$U = u(C_1) + \beta u(C_2)$$

where $u(C) = \log C$.

   (a) The consumer can borrow and lend at a given real interest rate r. Write down his life-time budget constraint, derive the optimum condition(s) and interpret the result. Solve for the levels of consumption and saving in period 1.

   (b) Discuss briefly the effects of an increase in the interest rate on consumption and saving in period 1.

   (c) Now assume that because of ageing the consumer has considerably lower labor income in period 2 than in period 1. However, the government has instituted a compulsory pension scheme. This means that when young the consumer has to pay a contribution τ to the pension system while when old he receives a pension $P = \tau(1 + r)$. How will this affect his consumption in the two periods and his personal saving in period 1? Can you think of any good economic reasons for introducing a pension scheme like this?

   (d) Suppose that the contribution to the pension system is still τ, while thepension is $P = (1 + \tilde{r})\tau$, where $\tilde{r}$ is equal to the sum of the growth rates of population and income. Explain why this may be an interesting case to look at. Discuss how consumption and personal savings in period 1 are affected by this pension system.

2. We now turn from the individual to the economy as a whole. The economy is populated by overlapping generations of identical individuals that live for two periods. Each generation is of the same size. Individuals supply one unit of labor when young, none when old. There is no pension scheme. The utility functions are as above. The "per-capita" production function is

$$y_t = f(k_t) = k_t^\alpha$$

where y_t is output per worker, k_t is capital per worker. The wage rate, w_t, and the real interest rate, r_t, are determined by the marginal productivity conditions

$$\begin{aligned} w_t &= f(k_t) - k_t f'(k_t) \\ r_{t+1} &= f'(k_t) \end{aligned}$$

(r_{t+1} is the interest rate on an investment from period t to period $t+1$).

   (a) Explain why in this economy the total capital stock in period $t+1$ is equal to the savings of the young in period t.

   (b) Use your results from questions 1 and 2 above to derive the law of motion for the capital stock (the equation that determines the evolution of the capital stock over time).

   (c) Define a steady state in this economy and explain how the capital stock in the steady state is determined.

2 Debt crisis

1. Next we look at the debt crisis model from Romer, chapter 12 (section 10). The government needs to issue debt D, for which it offers an interest rate factor R (gross interest rate). Investors are risk neutral, and can alternatively invest in a risk-free asset yielding $\bar{R}$. The government's ability to pay the investors RD in next period depends on its tax revenue, T. T is a random variable with cdf $F(\bullet)$. If $T \geq RD$, the government repays, but it defaults on the entire debt if $T < RD$. Let π denote the investors' perceived probability of default.

   (a) Interpret the equilibrium condition $(1-\pi)R = \bar{R}$

   (b) Interpret the equilibrium condition $\pi = f(RD)$

2. Let R_U and R_L be the levels of R such that $f(R_U D) = 1$ and $f(R_L D) = 0$. Assume that initially, $R_L < \bar{R} < R_U$. Use a graph to illustrate:

   (a) The equilibrium/equilibria of the model

   (b) Why the 'normal' and 'crisis' states are stable equilibria

   (c) The effect of shifts in $\bar{R}$

   (d) The effect of shifts in D