Exercises to Seminar 3
ECON 4310

September 20, 2013

1 Oil money

We consider a model for a small open economy. It is inhabited by a representative agent (of constant size) with utility function

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t, g_t)$$

where c_t is private consumption and g_t is public consumption. Let

$$u(c_t, g_t) = \log(c_t) + \theta \log(g_t)$$

The country has unlimited access to borrowing and lending in the international credit market at a constant interest rate r. For simplicity we assume

$$\beta(1 + r) = 1$$

(Make sure to use this later, since it simplifies many of your answers.) The agent has initial assets a_0, and recieves income w_t every period. Income grows at a constant rate z, such that

$$w_t = (1 + z)^t w_0$$

where w_0 is given. We assume that $z < r$. Its period t budget constraint is

$$c_t + a_{t+1} = w_t - \tau_t + (1 + r)a_t$$

with τ_t refering to a lump-sum tax collected by the government. The government faces a budget constraint

$$g_t + b_{t+1} = \tau_t + (1 + r)b_t$$

b_t is the government assets, and $b_0 = 0$ (no initial assets). We assume that no-Ponzi condition hold for both the consumers and government.

1. Write down the social planner's problem and find the first-order conditions for private consumption, public consumption, and assets.

2. Combine the first-order conditions with the intertemporal budget constraint of the agent to find the optimal path of private and public consumption.

3. What is the optimal sequence of taxes, $\{\tau_t\}_{t=0}^{\infty}$? (*Hint:* Think before you do any math)

4. Then assume that the government finds oil in period 0. This amounts to an increase in intitial assets b_0 by X. How does this affect the optimal path of private and public consumption?

5. Imagine that the state could hand out the entire oil revenues to the agent instead of keeping the income itself. Would this affect private or public consumption? (*Hint:* Think before you do any math)

2 Oil money and tax smoothing

Let us keep most of the set-up from the first problem. However, now the agent has preferences over consumption c and labor supply h. For simplicity, we assume that the wage rate is constant ($z = 0$). Preferences are given by

$$u(c_t, h_t) = \log(c_t - 0.5h_t^2)$$

Letting w_t be the per-hour wage, income is now $w_t h_t$. Further, we assume that lump-sum taxes are no longer possible. Instead, the government can only use a proportional tax on income. This gives tax revenues in period t as

$$tax_t = \tau_t w_t h_t$$

Taxes are raised to finance some (for now exogenous) expenditure of g every period.

1. What is the optimal labor supply in period t given a wage rate w_t and a tax rate τ?

2. Use labor supply to find the total tax revenue (tax_t) in period t for any tax rate τ. Illustrate tax revenues as a function of τ between zero and one.

3. It is clear that welfare of the agent is increasing in the net present value (NPV) of income. The optimal path of tax rates will therefore maximize the NPV of income subject to the restriction that the NPV of tax income must equal E. Write down this problem and find the first-order conditions. Confirm that the optimal tax rate is constant.

4. Describe what a constant tax rate implies for the level of tax_t, i.e. the taxes raised in every period, when the NPV of tax income is required to equal E.

5. Assume once more that the government receives oil income X in period 0. How does this affect tax_t? Can this be interpreted as a "fiscal rule"?

6. When both the tax rate and government spending is constant and initial assets b_0 are zero, we know that the government ran a balanced budget (i.e. $g_t = tax_t$) in every period prior to finding oil. Calculate how the stock of government assets, b_{t+1}, evolves after finding oil (i.e. after b_0 jumps to X).

7. Then assume that both w_t and g_t grow at a constant rate z, meaning

$$\begin{aligned} w_t &= (1+z)^t w_0 \\ g_t &= (1+z)^t g_0 \end{aligned}$$

   Assume also that the utility function changes to

$$u(c, h, t) = \log(c - 0.5(1+z)^t h^2)$$

   Re-calculate the labor supply, and confirm that it is (almost) the same as before (except that w_t is replaced by w_0)

8. Update your expression for tax revnues and the NPV of income. Confirm that the optimal tax rate still is constant.

9. Find tax_t for the optimal tax rate and compare tax revenues in every period before and after oil is found.

10. What is now the evolution of b_{t+1}? (We will still have balanced budget every period when $b_0 = 0$)

11. How do the fiscal rules derived in this problem set compare with the Norwegian fiscal rule for oil revenues?