Handout to Seminar 3
ECON 4310

October 9, 2013

Oil money and tax smoothing

Subquestion 8

As we saw in the seminar, tax revenues in period t are now

$$tax_t = \tau_t w_t h_t = \tau_t (1-\tau_t) w_0 w_t = (1+z)^t \tau_t (1-\tau_t) w_0^2$$

so they are increasing with the wage rate for a given tax rate. The optimal tax rate is found by solving

$$\max_{\{\tau_s\}_{s=0}^{\infty}} \sum_{t=0}^{\infty} \frac{(1+z)^t (w_0(1-\tau_t))^2}{(1+r)^t}$$

$$s.t.$$

$$\sum_{t=0}^{\infty} \frac{(1+z)^t \tau_t (1-\tau_t) w_0^2}{(1+r)^t} = E$$

First-order condition:

$$-2(1-\tau_t)\left(\frac{1+z}{1+r}\right) w_0^2 - \lambda \left(\frac{1+z}{1+r}\right) w_0^2 (1-2\tau_t) = 0$$

Can be simplified to:

$$\frac{1-\tau_t}{1-2\tau_t} = -\lambda$$

Since this must hold for all t, this once again implies a constant tax rate.

Subquestion 9

Tax revenues must still equal the NPV of expenditure less of any oil revenues. Let us see what that implies. Taxes in any period t are given by

$$tax_t = (1+z)^t \tau(1-\tau) w_0^2$$

or

$$tax_t = (1+z)^t tax_0$$

So in the intertemporal BC for the government we have:

$$\sum_{t=0}^{\infty} \frac{tax_t}{(1+r)^t} = E$$

or

$$\sum_{t=0}^{\infty} \left(\frac{1+z}{1+r}\right)^t tax_0 = E$$

Solving for the geometric sum we find

$$tax_0 = \frac{r-z}{1+r} E$$

Let us then once more increase b_0 by X, which reduces E by $(1+r)X$. Effect on taxes:

$$\Delta tax_0 = -(r-z)X$$

and

$$\Delta tax_t = -(r-z)(1+z)^t X$$

Taxes are therefore cut at an increasing rate – the largest tax cuts are reserved for the future. Why? Since we maintain a constant tax rate to smooth the cost from taxation.

Subquestion 10

What is now the evolution of b_{t+1}? Well, from the government BC we have $b_{t+1} = (1+r)b_t + tax_t - g_t$. Prior to finding oil, tax_t and g_t grow at the same rate, so there is always a balanced budget. After finding oil:

$$b_1 = (1+r)X - (r-z)X = (1+z)X$$

while for b_2 we find:

$$b_2 = (1+r)b_1 - (r-z)(1+z)X = [1+r-(r-z)](1+z)X = (1+z)^2 X$$

Hence we see that

$$b_{t+1} = (1+z)^{t+1} X$$

The government is actually saving more and more. The limit NPV of future assets is still converging – since $z < r$. The tax revenues are designed s.t. the growth rate of the oil fund equals the growth rate of the economy! Spend only $r - z$, not r (the factor of the fund at time t being spent on tax cuts)!

Subquestion 11

How do the fiscal rules derived in this problem set compare with the Norwegian fiscal rule for oil revenues? Most similar to the models that do not take growth into account (although we get some of the growth-elements when the oil fund is growing, but that is more motivated by adjustment). Why hard to tuse the last rule? Inter-generational distribution.