Exercises to Seminar 5
ECON 4310

October 30, 2013

1 Saving and risk

Consider an economy with constant population, $N_t = N$. The utility function of an agent who is young in period t is

$$u_t = \ln(c_{y,t}) + E_t \ln(c_{o,t+1})$$

where $c_{y,t}$ and $c_{o,t+1}$ are his consumption when young and old. There is no real capital in the economy. One unit of labor produces one unit of goods. Young agents are always endowed with $\omega_{y,t} = 1$ units of labor. The endowments of old agents, $\omega_{o,t+1}$ are uncertain. Half of them get $\omega_{o,t+1} = 1 + \Delta$, the other half $\omega_{o,t+1} = 1 - \Delta$. Individuals learn which group they belong to at the start of the period when they are old. They have no indication before that. Thus, ex ante, all agents are identical. There is only one asset, private lending with an interest rate r_{t+1} from period t to $t + 1$. No insurance against endowment risk is sold. Note that there is no aggregate risk.

1. Write down the period by period budget constraints for an agent that is young in period t. Let the assets that a representative young consumer carries over from period t to period $t + 1$ be denoted a_{t+1}.

2. Derive the first order conditions for maximum utility.

3. What is the equilibrium level of a_{t+1}? (No algebra needed).

4. Use the answers to the two preceding questions to determine the interest rate r_{t+1}

5. How does the interest rate and the equilibrium amount of saving depend on the degree of income risk? How do you explain what goes on here in relation to common theories about saving?

6. Would you get the same result if the utility function were quadratic?

2 Risky assets

We are looking at a consumer who lives for two periods. He starts life with no assets and leaves no bequests. His consumption in the two periods is C_0 and C_1. His earned income is Y_0 in the first period and Y_1 in the second period. In the first period he has the opportunity to invest in two assets which yield gross returns in the second period $1 + r_a$ and $1 + r_b$. The amount he invests in the first asset is A_a, in the second A_b. Y_1, r_a and r_b are stochastic.

1. Write down the budget equations for the consumer for the two periods.

2. The consumer's utility function is

$$U = u(C_0) + \frac{1}{1+\rho} E_0 u(C_1)$$

where ρ is the discount rate and

$$u(C) = \frac{C^{1-\theta}}{1-\theta}$$

Derive the first-order conditions for maximum utility and give a verbal interpretation of them.

3. Assume (for this question only) that the two assets have the same expected return. Show that the first-order conditions in this case imply that

$$cov(u'(C_1), 1 + r_a) = cov(u'(C_1), 1 + r_b)$$

Which asset will be most in demand, a or b?

4. Suppose the consumer earns his income from farming and that one of the assets is shares in a food-processing firm that buys its raw materials from his and similar farms. Should he buy shares in this firm and how much?

5. Assume now that asset a is risk-free. How does this change the first-order conditions?

6. Now, assume that the economy is inhabited by a large number of consumers identical to the one we have studied. Explain how the first-order conditions from question 5 can then be used to determine the expected excess return on the risky asset.