Exercises to Seminar 6, ECON 4310

November 13, 2013

1 Search models

This problem deals with the two-sided search model from lecture #17. Assume that the matching function is

$$m_t = m(u_t, v_t)$$

where m_t is the number of matches, while u_t and v_t are the number of unemployed and vacancies, respectively (the mass of workers and firms is set to unity). As in class, we define $p(\theta) = m(1, \theta)$ and $q(\theta) = m(\frac{1}{\theta}, 1)$, where $\theta = v/u$ is a measure of labor market tightness.

1. Assume that workers have the utility function $\sum_{t=0}^{\infty} \beta^t c_t$ and no saving opportunities. Let $V_e(w)$ and V_u denote the value functions for an employed and unemployed worker, respectively. Interpret the Bellman equation

$$V_e(w) = \beta \left[w + (1-\delta) V_e(w) + \delta V_u \right] \tag{1}$$

   where δ is the exogenous separation rate and w is the wage earned.

2. In this setup job separation happens with an exogenous probability δ. Assume that we make it possible for a worker to quit her job. If she quits, she enters unemployment next period, making it possible to search for a new job. Will she ever wish to do so in steady state?

3. Firms have the utility function $\sum_{t=0}^{\infty} \beta^t (\pi_t - x_t)$, where π_t is profits and x_t is the costs of having a vacant position. π_t is equal to y if the firm has one employee (it cannot have more) and zero otherwise. The value functions for a firm with a worker or a vacancy are given by $J_e(y - w)$ and J_v, respectively. Interpret the steady state Bellman equation:

$$J_v = \beta \left[-k + q(\theta) J_e(y - w^{ss}) + (1 - q(\theta)) J_v \right] \tag{2}$$

   where k is the per-period cost of a vacant position.

4. There is free entry for firms. What is the steady state value of J_v?

5. If we specify two extra Bellman equations:

$$V_u = \beta\left[b + p(\theta)V_e(w^{ss}) + (1 - p(\theta))V_u\right] \tag{3}$$

$$J_e(y - w) = \beta\left[y - w + (1 - \delta)J_e(y - w) + \delta J_v\right] \tag{4}$$

   as well as the bargaining solution

$$V_e(w) - V_u = \alpha S \tag{5}$$

   where S is the *surplus*:

$$S = V_e(w) + J_e(y - w) - V_u - J_v \tag{6}$$

   we can write the model more compactly in terms of only S and θ. The two equations that define equilibrium are in this case

$$S = \frac{k}{(1 - \alpha)q(\theta)} \tag{7}$$

$$S = \frac{y - b}{\rho + \delta + \alpha p(\theta)} \tag{8}$$

   Draw a diagram that illustrates how the steady state values of S and θ are determined.

6. Combine (7) and (8) to get an equation that only depends on θ (and exogenous parameters). Show analytically that

$$\frac{d\theta}{dy} > 0$$

   Also illustrate it in your diagram.

7. Describe the flows in and out of unemployment in this model. Write down the law of motion for u_t.

8. Use the law of motion to find steady state unemployment, and show how it falls when y is increased.

9. Then use the fact that $v = \theta u$. Show that steady state v increases when y goes up.

10. Combine the steady state restriction on J_v from 4. with equation (4) as well as the bargaining solution (5)-(6) to obtain an expression for the steady state wage rate.

11. RBC models predict a procyclical real wage, but in the data it appears to be acyclical or moderately countercyclical. What is the prediction from our search model? (*Here you should only discuss, not derive anything. If you want to get an analytical result, think about the initial increase in S from a higher y in (8), and see what the wage becomes if this is the full response. Then look at the diagram to see if this over- or underestimates the change in S.*)

12. The effect of a higher y and a lower b are the same (since only $y - b$ matters). Explain briefly the intuition for why higher benefits increase unemployment. Compare with the prediction from the Shapiro-Stiglitz model. (Explain the basic intuition, do not derive anything.)

13. Explain what the Beveridge curve is (short), and specify which equation that defines the Beveridge curve in our model.

14. During the financial crisis, we've witnessed a shift in the Beveridge curve such as in Figure 1. Explain why a change in y cannot rationalize that in our model.

15. Try to show that a change in δ might explain the shift. Give an interpretation. (*Hint: You should also think about how this affects θ.*)

16. Do you think this is a reasonable explanation?

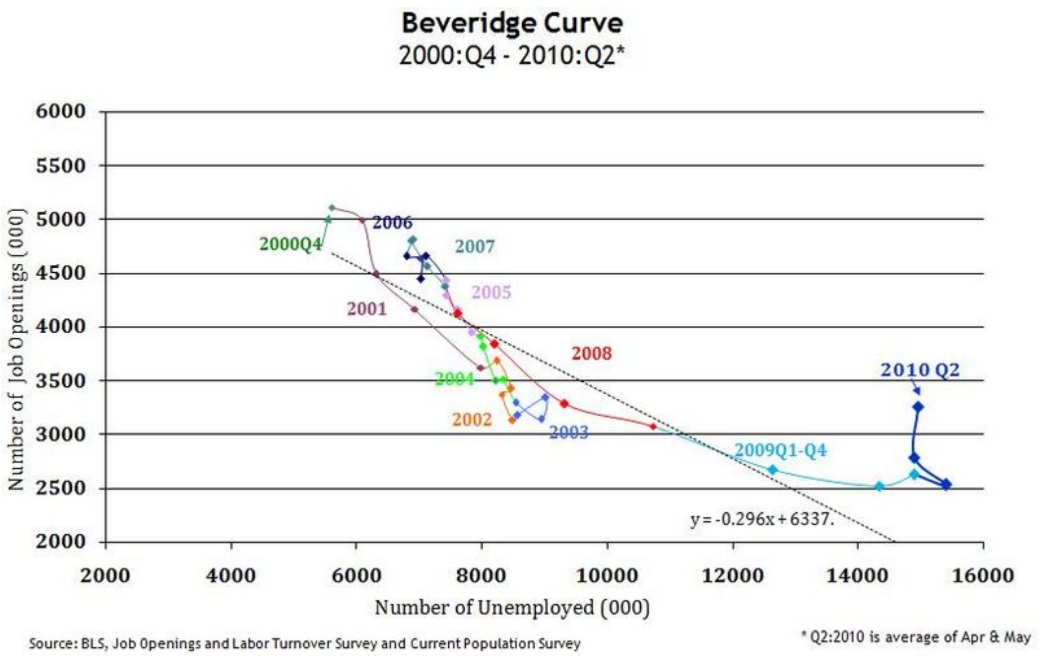

Figure 1: Shift in the Beveridge curve