Answers (Sigurd)

1 Search models

This problem deals with the two-sided search model from lecture #17. Assume that the matching function is

$$m_t = m(u_t, v_t)$$

where m_t is the number of matches, while u_t and v_t are the number of unemployed and vacancies, respectively (the mass of workers and firms is set to unity). As in class, we define $p(\theta) = m(1, \theta)$ and $q(\theta) = m(\frac{1}{\theta}, 1)$, where $\theta = v/u$ is a measure of labor market tightness.

1. Assume that workers have the utility function $\sum_{t=0}^{\infty} \beta^t c_t$ and no saving opportunities. Let $V_e(w)$ and V_u denote the value functions for an employed and unemployed worker, respectively. Interpret the Bellman equation
$$V_e(w) = \beta \left[w + (1 - \delta) V_e(w) + \delta V_u \right] \tag{1}$$
where δ is the exogenous separation rate and w is the wage earned.

    - **Solution:** $V_e(w)$ and V_u denote, respectively, the value of being employed (at wage w) and unemployed at the end of the period. If you are employed at the end of the period, then next period you work and consume w and then suffer a separation with probability δ and become unemployed, or continue to work at wage w with probability $1 - \delta$.

2. In this setup job separation happens with an exogenous probability δ. Assume that we make it possible for a worker to quit her job. If she quits, she enters unemployment next period, making it possible to search for a new job. Will she ever wish to do so in steady state?

    - **Solution:** No. If quitting a job with wage w was optimal in steady state it wouldn't be optimal to accept the job at wage w either. And we know from the Nash bargaining that $V(w) > V_u$, so quitting the job is never optimal.

3. Firms have the utility function $\sum_{t=0}^{\infty} \beta^t(\pi_t - x_t)$, where π_t is profits and x_t is the costs of having a vacant position. π_t is equal to y if the firm has one employee (it cannot have more) and zero otherwise. The value functions for a firm with a worker or a vacancy are given by $J_e(y-w)$ and J_v, respectively. Interpret the steady state Bellman equation:

$$J_v = \beta\left[-k + q(\theta)J_e(y - w^{ss}) + (1 - q(\theta))J_v\right] \tag{2}$$

where k is the per-period cost of a vacant position.

   - **Solution:** Value of a vacancy at the end of the period: Next period the firms pay k. With probability $(1 - q(\theta))$ the firm fails to meet a worker and continue the search. With probability $q(\theta)$ the firm meets a worker, and Nash bargaining induces both the firm and the worker to accept the match.

4. There is free entry for firms. What is the steady state value of J_v?

   - **Solution:** Firms must be indifferent between posting a vacancy or not. The alternative to posting a vacancy is to do nothing which has zero value. Hence, the steady state value of $J_v = 0$

5. If we specify two extra Bellman equations:

$$V_u = \beta\left[b + p(\theta)V_e(w^{ss}) + (1 - p(\theta))V_u\right] \tag{3}$$

$$J_e(y-w) = \beta\left[y - w + (1-\delta)J_e(y-w) + \delta J_v\right] \tag{4}$$

as well as the bargaining solution

$$V_e(w) - V_u = \alpha S \tag{5}$$

where S is the *surplus*:

$$S = V_e(w) + J_e(y-w) - V_u - J_v \tag{6}$$

we can write the model more compactly in terms of only S and θ. The two equations that define equilibrium are in this case

$$S = \frac{k}{(1-\alpha)q(\theta)} \tag{7}$$

$$S = \frac{y-b}{\rho + \delta + \alpha p(\theta)} \tag{8}$$

Draw a diagram that illustrates how the steady state values of S and θ are determined.

- **Solution:** The intersection between the upwards sloping (in θ) (7) and downwards sloping (8). Existence requires $k/(1-\alpha) < (y-b)/(\rho+\delta)$

6. Combine (7) and (8) to get an equation that only depends on θ (and exogenous parameters). Show analytically that

$$\frac{d\theta}{dy} > 0$$

Illustrate it also in your diagram.

- **Solution:** Combining (7) and (8) gives

$$\begin{aligned} \frac{k}{(1-\alpha)q(\theta)} &= \frac{y-b}{\rho+\delta+\alpha p(\theta)} \\ &\Leftrightarrow \\ (\rho+\delta+\alpha p(\theta))\,k &= (y-b)(1-\alpha)q(\theta) \end{aligned}$$

Total differentiation with respect to y:

$$k\alpha p'(\theta)\frac{d\theta}{dy} = (1-\alpha)q(\theta) + (1-\alpha)(y-b)q'(\theta)\frac{d\theta}{dy}$$

or

$$\frac{d\theta}{dy} = \frac{(1-\alpha)q(\theta)}{(k\alpha p'(\theta) - (1-\alpha)(y-b)q'(\theta))}$$

The term in the denominator is > 0 since $q' < 0$. Hence $d\theta/dy > 0$. Illustrate in a diagram? Shifts the 'supply' curve up.

7. Describe the flows in and out of unemployment in this model. Write down the law of motion for u_t.

- **Solution:** There are $\delta(1-u_t)$ new unemployed every period. But $p(\theta)u_t$ exit unemployment every period as well. Hence the law of motion is

$$u_{t+1} = u_t + \delta(1-u_t) - p(\theta)u_t$$

8. Use the law of motion to find steady state unemployment, and show how it falls when y is increased.

- **Solution:** Set $u_{t+1} = u_t = u$ in the law of motion. Gives

$$u = \frac{\delta}{\delta + p(\theta)}$$

Effect from changes in y? Differentiate with respect to θ:

$$\frac{du}{d\theta} = -\frac{\delta}{(\delta + p(\theta))^2} p'(\theta) < 0$$

So u is decreasing in θ. And θ is increasing in y!

9. Then use the fact that $v = \theta u$. Show that steady state v increases when y goes up.

   - **Solution:** insert for steady state unemployment to get

$$\begin{aligned} v &= \theta \frac{\delta}{\delta + p(\theta)} \\ &= \frac{\delta}{\frac{\delta}{\theta} + \frac{p(\theta)}{\theta}} \\ &= \frac{\delta}{\frac{\delta}{\theta} + q(\theta)} \end{aligned}$$

   The denominator is decreasing in θ since $q' < 0$, hence $dv/d\theta > 0$

   - An increase in y increases the total surplus from a match, making it more valuable to post vacancies, so v and θ increases. This leads to a fall in unemployment since the job finding rate increases. Increase in productivity makes unemployment and vacancies move in opposite directions.

10. Combine the steady state restriction on J_v from 4. with equation (4) as well as the bargaining solution (5)-(6) to obtain an expression for the steady state wage rate.

    - **Solution:** Start out with (4) and impose $J_v = 0$:

$$J_e(y - w) = \beta(y - w + (1 - \delta)J_e(y - w))$$

Use $\beta = 1/(1+\rho)$ to get

$$J_e(y-w)(\rho+\delta) = y - w$$

and then use $J_e(y-w) = (1-\alpha)S$

$$(\delta+\rho)(1-\alpha)S = y - w$$

or

$$w = y - (\rho+\delta)(1-\alpha)S$$

11. RBC models predict a procyclical real wage, but in the data it appears to be acyclical or moderately countercyclical. What is the prediction from our search model?

    - **Solution:** First: Clarify what we mean by procyclical: y up implies w up. Higher y increases the surplus S from a match and the labor market tightness θ. The wage rate increases since the bargaining solution implies a fixed share α of the surplus to the worker. why? Suppose nothing happens to wages. Since θ increases the worker's utility gain (surplus) from working $V_e - V_u$ goes down since the worker now is more likely to find a job when unemployed. Therefore wages must go up to make the utility gain from working increase. How do we show it? Here are two ways

      (a) Notice that we can express the worker's surplus in two ways. Combining the Bellman equations (1) and (3) we get

      $$V_e - V_u = \frac{w-b}{\rho+\delta+p(\theta)}$$

      and using the solution to the Nash bargaining problem we get

      $$V_e - V_u = \alpha S$$

      The bargaining solution tells us that the workers surplus increases, while the Bellman equations tells us that the increase must be due to higher wages since higher θ reduces the worker's surplus $V_e - V_u$.

(b) Look at equation (8).

$$S = \frac{y-b}{\rho+\delta+\alpha p(\theta)}$$

Suppose that nothing happens to θ. The total effect on equilibrium S is given from equation (8), i.e. assume nothing happens to θ. Then

$$\frac{dS}{dy} = \frac{1}{\rho+\delta+\alpha p(\theta)}$$

and

$$\frac{dw}{dy} = 1-(\rho+\delta)(1-\alpha)\frac{1}{\rho+\delta+\alpha p(\theta)} = \frac{\alpha(p(\theta)+\rho+\delta)}{\rho+\delta+\alpha p(\theta)} > 0$$

So if nothing happens to θ, the wage rate increases. But θ will increase, so the equilibrium effect on S is over-estimated above, which means that the equilibrium wage will increase even more.

12. The effect of a higher y and a lower b are the same (since only $y-b$ matters). Explain briefly the intuition for why higher benefits increase unemployment. Compare with the prediction from the Shapiro-Stiglitz model. (Explain the basic intuition, do not derive anything.)

- **Solution:** Higher benefits increase unemployment since workers now demand (and manage to get) a higher wage. Reduces firms' surplus from a match, lowering the number of vacancies, and thus increases unemployment. What about Shapiro-Stiglitz? Here the effect is that higher benefits makes the threat of being fired less severe. Hence necessary to have a higher wage in equilibrium to avoid shirking, which reduces employment.

13. Explain what the Beveridge curve is (short), and specify which equation that defines the Beveridge curve in our model.

- **Solution:** Gives the equilibrium relationship between vacancies and unemployment. Higher value of v requires a lower value of

u, since the higher v increases labor market tightness, making it more likely to get a job, increasing the flow from unemployment to employment. The curve is defined by the steady state value of unemployment when you insert for $\theta = v/u$.

$$u = \frac{\delta}{\delta + p(v/u)}$$

14. During the financial crisis, we've witnessed a shift in the Beveridge curve such as in Figure 1. Explain why a change in y cannot rationalize that in our model.

    - **Solution:** A drop in y gives a lower θ. Changes in u and v are traced out by moving along the Beveridge curve. We do get higher unemployment, but the BC does not shift.

15. Try to show that a change in δ might explain the shift. Give an interpretation. (*Hint: You should also think about how this affects θ.*)

    - **Solution:** a higher δ shifts the Beveridge curve out (higher unemployment for every value of v). In addition, the labor market tightness θ is reduced and we get higher equilibrium unemployment.

16. Do you think this is a reasonable explanation?

    - **Solution:** currently we are in a situation with high unemployment, despite high vacancy. Before the financial crisis, unemployment would have been much lower at this vacancy. What do you think is most reasonable? That the unemployment is high because it is now more likley to lose a job, or that is high because it is more difficult for unemployed workers and firms to match? The latter would be a change in the matching technology, for instance due to a mismatch between workers skills and skills required by vacant firms.
    - chek out this: http://economix.blogs.nytimes.com/2013/03/07/an-odd-shift-in-an-unemployment-curve/?_r=1

- Regarding increased separation rates, the data does not seem to support this story.[1]

[1] http://www.richmondfed.org/publications/research/working_papers/2013/pdf/wp13-16.pdf
(p. 7) argues that separation rate actually fell over the course of the Great Recession.