

How much should the nation save?

Econ 4310 Lecture 2

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Outline

- Golden Rule
- Social planner's choice
- Market solution
- Bringing back natural growth

Slide set covers first two, lecture may go further

The Golden Rule of Accumulation

Consumption per efficiency unit of labor in steady state is:

$$c = f(k) - \gamma k \quad (1)$$

First order condition for maximum is $f'(k) - \gamma = 0$. Golden rule level of k , k^{**} is determined by

$$f'(k^{**}) = \gamma \quad (2)$$

$$r^{**} = \gamma$$

Interest rate equal to natural growth rate

Savings rate required to reach k^{**} :

$$s^{**} = \gamma k^{**} / f(k^{**}) = r^{**} k^{**} / f(k^{**})$$

Along the Golden rule path the savings rate equals the income share of capital.

If s is increased beyond s^{**} , consumption is reduced both now and in all future!

The Ramsey model, simple version

Social planner maximizing utility over all future generations given

- Zero population growth
- zero productivity growth

Welfare function

$$U_0 = u(c_0) + \beta u(c_1) + \beta^2 u(c_2) + \dots = \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

$0 < \beta < 1$ discount factor (subjective)

$\beta = \frac{1}{1 + \rho}$ $\rho =$ discount rate, degree of impatience

$u(C)$ period utility $u' > 0$ $u'' < 0$, $u'(0) = \infty$

The accumulation equation

$$y_t = f(k_t)$$

$$k_{t+1} = k_t + i_t$$

$$k_{t+1} = k_t + f(k_t) - c_t \Leftrightarrow c_t = f(k_t) + k_t - k_{t+1} \quad (2)$$

- y_t output
- k_t capital
- c_t consumption

The maximization problem

$$\max U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

given

$$c_t = f(k_t) + k_t - k_{t+1} \text{ and } k_t \geq 0, k_0 = \bar{k}_0$$

Insert for c_t from accumulation equation in utility function.

Solution

Utility function after insertions is :

$$U_0 = \dots + \beta^t u(f(k_t) + k_t - k_{t+1}) + \beta^{t+1} u(f(k_{t+1}) + k_{t+1} - k_{t+2}) + \dots$$

$k_t + 1$ appears only in terms t and $t + 1$

Necessary condition for maximum are:

$$\frac{\partial U_0}{\partial k_{t+1}} = -\beta^t u'(c_t) + \beta^{t+1} u'(c_{t+1})(1 + f'(k_{t+1})) = 0$$

or

$$u'(c_t) = \beta u'(c_{t+1})(1 + f'(k_{t+1})) \text{ for } t = 0, 1, 2, \dots, \infty \quad (3)$$

The Consumption Euler-equation

$$u'(c_t) = \beta u'(c_{t+1})(1 + f'(k_{t+1})) \text{ for } t = 0, 1, 2, \dots, \infty \quad (4)$$

- A marginal transfer of resources between periods t and $t + 1$ should not increase total welfare
- A marginal transfer of resources between any two periods should not increase total welfare

$$\frac{\beta u'(c_{t+1})}{u'(c_t)} = \frac{1}{1 + f'(k_{t+1})} \text{ for } t = 0, 1, 2, \dots, \infty \quad (5)$$

- Marginal rate of substitution should equal marginal rate of transformation

Problem not solved yet

Two difference equations:

$$k_{t+1} = f(k_t) + k_t - c_t \quad (6)$$

$$u'(c_{t+1}) = [\beta(1 + f'(k_{t+1}))]^{-1} u'(c_t) \quad (7)$$

Non-linear system in two unknown time series. One initial condition (k_0 given)

- Exclude paths where $k_t \rightarrow 0$ when $t \rightarrow \infty$ (resource constraint).
- Choose the path with highest utility among the remaining
- Closed form solutions only in special cases. Simulation usually necessary.

The stationary state

Definition:

$$k_{t+1} = k_t = k^* \text{ and } c_{t+1} = c_t = c^*$$

Insert in difference equations (6) and (7):

$$\begin{aligned} k^* &= f(k^*) + k^* - c^* \\ u'(c^*) &= [\beta(1 + f'(k^*))]^{-1} u'(c^*) \end{aligned}$$

Solution:

$$f'(k_*) = (1 - \beta)/\beta = \rho \tag{8}$$

$$c^* = f(k^*) \tag{9}$$

- Subjective discount rate determines capital intensity.
item Capital intensity determines output.
- No savings, consumption = output

Comparison to golden rule

Remember: No population growth, no productivity growth

- Golden rule: $f'(k^*) = 0$
- Ramsey rule: $f'(k^*) = \rho > 0$
- Social planner chooses less capital

Graphical solution

Phase diagram will be drawn to show:

- Social planner will choose a path that leads to the stationary state
- If $k_0 < k_*$, both consumption increases gradually as the economy moves towards the steady state.