1 Ricardian equivalence

- Consider a government which has a need for government spending (given exogenously) og $\{g_t\}_{t=0}^{\infty}$ (a sequence of wars, say). Initial government debt is zero.
- Assume government has commitment or, alternatively, that policies are time consistent:
  - Definition: a policy at time $t+k$ that seemed optimal at time t may must be optimal to carry out when period $t+k$ appears.
  - Examples where time consistency is violated: repeated elections (and possibly new government each period), crime and punishment.
  - Time consistency puts strong restrictions on future plans of the government.
- To finance the expenditures, the overnment can issue (one-period) debt b_t and issue lump-sum taxes T_t.
- No risk and no arbitrage means that the rate of return on bonds must equal the rate of return on capital, r_t. The government therefore faces a sequence of borrowing constraints

$$\begin{aligned} g_0 &= b_1 + T_0 \\ g_1 + (1+r_1)\, b_1 &= b_2 + T_1 \\ g_2 + (1+r_2)\, b_2 &= b_3 + T_2 \\ &\ldots \end{aligned}$$

  Suppose the government also faces a no-Ponzi scheme condition (always true in the Ramsey model, not always true in the Diamond OLG model):

$$\lim_{T\to\infty} \frac{b_T}{(1+r_1)(1+r_2)\cdot \ldots \cdot (1+r_T)} = 0$$

  Then the sequence of budget constraints can be written as a natural NPV condition where the present value of government expenditures equals the present value of taxes:

$$\sum_{t=0}^{\infty} p_t g_t = \sum_{t=1}^{\infty} p_t T_t$$

  where $p_0 = 1$ and p_t is the market discount factor

$$p_t = \frac{1}{(1+r_1)(1+r_2)\cdot \ldots \cdot (1+r_t)}$$

  Or, in terms of primary deficit,

$$\sum_{t=0}^{\infty} p_t (g_t - T_t) = 0$$

- Consider now the budget constraint for the individual households. Note: households face the same interest rates as the government:

$$\begin{aligned} c_0 + b_1 + k_1 &= (1 + r_0)\, k_0 + w_0 - T_0 \\ c_1 + b_2 + k_2 &= (1 + r_1)\,(k_1 + b_1) + w_1 - T_1 \\ c_2 + b_3 + k_3 &= (1 + r_2)\,(k_2 + b_2) + w_2 - T_2 \\ &\;\;\; ... \end{aligned}$$

  Given the no-Ponzi-scheme condition, the sequence of budget constraints can be written as a natural NPV condition where the present value of consumption equals the wealth plus the present value wages minus NPV of taxes:

$$\begin{aligned} \sum_{t=0}^{\infty} p_t c_t &= (1 + r_0)\, k_0 + \sum_{t=1}^{\infty} p_t\,(w_t - T_t) \\ &= (1 + r_0)\, k_0 + \sum_{t=1}^{\infty} p_t w_t - \sum_{t=1}^{\infty} p_t T_t \end{aligned}$$

- Use the government budget constraint to rewrite it:

$$\sum_{t=0}^{\infty} p_t c_t = (1 + r_0)\, k_0 + \sum_{t=1}^{\infty} p_t w_t - \sum_{t=1}^{\infty} p_t g_t.$$

- Conclusion: it is only the NPV of government expenditures that matters, not the timing of taxes. In fact, debt is irrelevant.

- This is the *Ricardian equivalence* result

- Intuition: government debt is not net wealth because government debt implies a future tax burden. When debt increases, housholds save so as to be able to pay the future debt

- Conditions necessary for Ricardian equivalence to hold:

  1. Taxes are lump sum (i.e., non-distortive)
  2. Households are infinitely-lived or, equivalently, households are finitely lived and
     (a) have altruism toward their children, so their preferences are given by
     $$u\,(c_t) + \beta V\,(k_{t+1})\,,$$
     where $u\,(c_t)$ is utility over own consumption and $V\,(k_{t+1})$ is the utility of the child (given an inheritance of k_{t+1} units of capital).

β is the weight on child's utility (altruistic parameter). Note that since

$$\begin{aligned} V(k_{t+1}) &= u(c_{t+1}) + \beta V(k_{t+2}) \\ V(k_{t+2}) &= u(c_{t+2}) + \beta V(k_{t+3}) \\ &\ldots \end{aligned}$$

which implies

$$V(k_0) = \sum_{t=0}^{T} \beta^t u(c_t) + \beta^T V(k_T),$$

so that if $\beta < 1$, then this is just the infinite-horizon model.

(b) there are no constraints on bequests (can give both negative and positive bequests)