1 Notes Introduction

1.1 Motivation

- Dynamic macro. Logic of course:
  - Start with frictionless economies. Then introduce frictions
  - As in medicine, start with analysis of healthy individuals. Afterwards, turn to study sick patients.
- Central tool: competitive equilibrium
  - powerful and simple (need not think of what could have happened, as in game theory)
  - Specify environment:
    1. Physical environment (preferences, endowments, technology
    2. Government (policies, taxes, laws)
    3. Markets (the key interaction between agents)
  - Solve for a competitive equilibrium: Given allocations, government policies, and prices:
    1. All agents and firms optimize
    2. All markets clear

1.2 A static model

- Physical environment
  - Preferences over consumption c and leisure l:

    $$u(c, l)$$

    where $\partial u/\partial c \equiv u_1 > 0$, $u_2 > 0$, u is twice differentiable and strictly concave. For simplicity (to avoid corner solutions), we assume

    $$\begin{aligned} \lim_{c \to 0} u_1 &= \infty \\ \lim_{c \to 0} u_l &= \infty. \end{aligned}$$

    There are N individuals, each have an equal amount of capital, k_0/N, which can be rented to firms. They also have one unit of leisure.
  - Technology: $\exists$ M firms, each operating a technology

    $$y = zf(k, n),$$

    where $f_1 > 0$, $f_2 > 0$, f is strictly quasiconcave, and f is homogeneous of degree one, i.e., *constant return to scale*:

    $$\lambda y = zf(\lambda k,)$$

for $\lambda > 0$. Moreover, *Inada conditions* hold:

$$\begin{aligned}\lim_{k\to 0} f_1 &= \lim_{l\to 0} f_2 = \infty \\ \lim_{k\to\infty} f_1 &= \lim_{l\to\infty} f_2 = 0\end{aligned}$$

- Markets: firms rent capital and labor on competitive markets. Firms sell output at a competitive market for consumption goods.
- Optimization:
  - Prices: The consumption good is the numeraire. Wage is w and rental rate of capital is r.
  - Consumer's problem: Take prices as given. Solve

    $$\max_{c,l,k_s} u\left(c,l\right)$$

    subject to

    $$\begin{aligned}c &\leq w\left(1-l\right)+rk_s & (1)\\ 0 &\leq k_s \leq \frac{k_0}{N} & (2)\\ 0 &\leq l \leq 1 & (3)\\ c &\geq 0 & (4)\end{aligned}$$

    Clearly, it is optimal to set $k_s = \frac{k_0}{N}$. Ignore case of $l = 1$ (since nothing would be produced). Properties of u ensure $c > 0$ and $l > 0$. Formulate problem as a Lagrangian problem:

    $$\Lambda = u\left(c,l\right)+\mu\left(w+r\frac{k_0}{N}-wl-c\right)$$

    Given properties of u, the optimum is unique and characterized by the FOC:

    $$\begin{aligned}\frac{\partial\Lambda}{\partial c} &= u_1-\mu = 0\\ \frac{\partial\Lambda}{\partial l} &= u_2-\mu w = 0\\ \frac{\partial\Lambda}{\partial \mu} &= w+r\frac{k_0}{N}-wl-c = 0\end{aligned}$$

    Substitute away c and μ and obtain

    $$wu_1\left(w+r\frac{k_0}{N}-wl,l\right)-u_2\left(w+r\frac{k_0}{N}-wl,l\right)=0,$$

    or

    $$w=\frac{u_2}{u_1}$$

    Figure 1.1

- Firm's problem: Take prices as given. Solve

$$\max_{k,n} \{zf(k,n) - rk - wn\}.$$

  Optimal allocation is the *marginal product conditions*:

$$\begin{aligned} zf_1 &= r \\ zf_2 &= w \end{aligned}$$

  Since f is homogeneous of degree one,

$$zf(k,n) = zf_1 k + zf_2 n,$$

  which implies that the firm profits are zero! This implies

  * Don't have to keep track of where profits go
  * If k^* and n^* are optimal choices, then

$$zf(k^*, n^*) - rk^* - wn^* = 0,$$

    so the optimal scale of a firm is indeteminate. Don't have to keep track of the number of firms (could set $M = 1$)

- Competitive equilibrium is an *allocation* $\{c, l, k, n\}$ and a set of *prices* $\{r, w\}$ such that

  1. Consumers choose c and l optimally, given (r, w)
  2. Representative firm chooses k and n optimally, given (r, w)
  3. Markets clear

  - Market clearing requires supply=demand in all markets:

$$\begin{aligned} N(1-l) &= n \\ y &= Nc \\ k_0 &= k \end{aligned}$$

    Total value of excess demand across markets is

$$nc - y + w(n - N(1-l)) + r(k - k_0)$$

    This expression is ZERO from the consumers' budget constraint
  - Walras' law: need only two market-clearing conditions
  - Drop condition $y = Nc$. Have five unknowns (l, n, k, w, r) and five equilibrium conditions (note: ignore the number of consumers and firms, N and M). Substitute to obtain one equation in one unknown l:

$$zf_2 \cdot u_1(zf(k_0, 1-l), l) - u_2(zf(k_0, 1-l), l) = 0,$$

    and given l we solve for r, w, n, k, c.

- Pareto optimality
  - Pareto optimality is an allocation such that no individual can be made better off without anyone else being made worse off
  - Focus on equally weighted fictitious social planner allocation:

$$\begin{aligned} & \max u(c,l) \\ & \text{subject to} \\ c &= zf(k_0, 1-l) \end{aligned}$$

    Solution is

$$zf_2 \cdot u_1(zf(k_0, 1-l), l) - u_2(zf(k_0, 1-l), l) = 0,$$

    i.e., the same as before! Figure 1.2

    1. *First welfare theorem*: If there are no externalities and markets are complete, then a competitive equilibrium allocation is Pareto optimal
    2. *Second welfare theorem*: A Pareto optimal allocation can be supported as a competitive equilibrium given some tranfers.
  - Welfare theorems are useful for solving for competitive equilibria

- Example

$$\begin{aligned} u(c,l) &= \frac{c^{1-\gamma}-1}{1-\gamma} + l \\ f(k,n) &= k^{\alpha} n^{1-\alpha} \end{aligned}$$

  Planner problem is

$$\max_l \left\{ \frac{\left[zk_0^{\alpha}(1-l)^{1-\alpha}\right]^{1-\gamma} - 1}{1-\gamma} + l \right\}$$

  Solution is

$$\begin{aligned} n &= 1-l = \left[(1-\alpha)\left(zk_0^{\alpha}\right)^{1-\gamma}\right]^{\frac{1}{\alpha+(1-\alpha)\gamma}} \\ &\Rightarrow \\ c &= \left[(1-\alpha)^{1-\alpha}\left(zk_0^{\alpha}\right)\right]^{\frac{1}{\alpha+(1-\alpha)\gamma}} \\ w &= \left[(1-\alpha)^{1-\alpha}\left(zk_0^{\alpha}\right)\right]^{\frac{\gamma}{\alpha+(1-\alpha)\gamma}} \end{aligned}$$

  Note: c and w are increasing in z. But effect on l is ambiguous: $\partial l / \partial z < 0$ iff $\gamma < 1$.

- Government
  - Assume a government must provide a quantity g of a public good, financed by lump-sum taxes τ. Budget must balance:

    $$g = \tau$$

    Preferences are $u(c, l) + v(g)$. Ignore v since g is exogenous.
  - Assume that labor is the only factor of production:

    $$y = zn$$

  - Optimization problem is

    $$\begin{aligned} & \max u(c, l) \\ & \text{subject to} \\ c &= w(1 - l) - \tau \end{aligned}$$

    FOC is, as before,

    $$-wu_1 + u_2 = 0$$

  - Firm's problem is

    $$\max_n \{n(z - w)\},$$

    i.e., infinitely elastic labor demand at wage $w = z$.
  - Competitive equilibrium conditions: same as before, plus government budget clearing.
  - Use a planner problem to solve for the c.e.:

    $$\begin{aligned} & \max_{c,l} u(c, l) \\ & \text{subject to} \\ c + g &= z(1 - l) \end{aligned}$$

    which implies a FOC

    $$-zu_1(z(1 - l) - g, l) + u_2(z(1 - l) - g, l) = 0$$

  - Figure 1.4. Note that the balanced budget multiplier is less than one:

    $$\frac{\partial y}{\partial g} < 1$$