# Problem Set 0: Introduction

## Exercise 0.1: Immigration in the Solow model

Consider a closed economy with a neoclassical production function, exogenous technological progress, $A_t$, a fixed saving rate, $s$, and a constant labor force, $L$, as described by the following equations (the Solow model)

$$K_{t+1} - K_t = sY_t - \delta K_t \tag{1}$$
$$Y_t = K_t^{\alpha}(A_t L)^{1-\alpha},\ 0 < \alpha < 1, \tag{2}$$
$$A_{t+1} = (1+g)A_t,\ A_0 > 0,$$

where $0 \leq \delta \leq 1$ is the depreciation rate of physical capital (in the lecture we have abstracted from depreciation for simplicity).

(a) Remove the trend from Equations (1) and (2) by writing all endogenous variables $X_t$ in terms of efficiency units $x_t \equiv X_t/(A_t L)$.

(b) Compute the wage rate, $w_t$, and the rental rate of capital, $r_t$, (the interest rate of this economy will be $r_t - \delta$) in this economy.

(c) Compute the steady-state capital stock per efficiency unit, $k^\star > 0$, of this economy.

(d) Show that the capital stock per efficiency unit, $k_t$, is increasing over time as long as $0 < k_t < k^\star$ (hint: look at the ratio $k_{t+1}/k_t$ in the capital accumulation equation to answer this question).

(e) Suppose the economy is in a steady-state. What happens to the aggregate output, the wage rate, and the rental rate on impact if the number of workers in the economy increases by $\partial L$ due to immigration?

(f) What happens to aggregate output, the wage rate, and the rental rate over time after the immigration wave?

(g) Would the wage rate be higher in the long-run if there had been no immigration?

## Exercise 0.2: A static competitive equilibrium

Consider a static economy with a representative consumer that has the following preferences over consumption, $c$, leisure, $l$, and a public good, $g$,

$$u(c, n, g) = \log\left(c - \psi(1-l)^{\theta}\right) + \log(g), \quad \theta > 1,$$

and is subject to the budget constraint

$$c = (1-\tau^n)(1-l)w + rk_0,$$

where $\tau^n$ is a proportional tax rate on labor income, and $k_0$ denotes the consumers' initial endowment of physical capital. The representative firm produces consumable output, $y$, with the following technology

$$y = k^\alpha n^{1-\alpha}, \quad 0 < \alpha < 1,$$

by renting physical capital, $k$, from consumers at the rental rate $r$, and labor, $n$, at the wage rate $w$. The government spends a fixed fraction $\gamma = g/y$ of output on public goods by setting labor income taxes to balance the government's budget

$$\tau^n n w = g.$$

Remember the definition of a competitive equilibrium:

> A **competitive equilibrium** is an allocation $\{c, l, k, n\}$ and a set of prices and taxes $\{r, w, \tau^n\}$ such that
>
> (1) The representative consumer chooses $c$ and $l$ to maximize utility subject to the private budget constraint, taking as given prices and taxes.
>
> (2) The representative firm chooses physical capital $k$ and labor $n$ to maximize profits, taking as given prices.
>
> (3) The government chooses tax policy $\tau^n$ to balance the government budget.
>
> (4) The markets for goods, capital, and labor clear.

In what follows, we will compute the competitive equilibrium of this economy step by step.

(a) Solve the consumers maximization problem.

(b) Derive the optimality conditions of the firm's maximization problem.

(c) Find the equilibrium prices $\{r, w\}$ that clear the labor and the capital market for a given tax rate $\tau^n$.

(d) Show that the wage income of the consumer is equal to a constant share of output, $wn = (1-\alpha)y$. What is the equilibrium tax rate $\tau^n$ of this economy that balances the government budget?

(e) Verify that Walras' law holds in the competitive equilibrium derived above, i.e., check whether the goods market clears at equilibrium prices and taxes

$$y(\tau^n) = c(\tau^n) + g.$$