Problem Set 4:
Optimal Fiscal Policy

Exercise 4.1: The Norwegian Handlingsregelen

Consider a small open economy populated with non-overlapping generations of households that live for one period. The size of each generation is one, and the generation living in period t earns an exogenously given wage w_t. The government of the economy is endowed with initial resources (due to an oil windfall, for example) of value

$$B = -b_0$$

where b_0 denotes the initial debt position of the government as in previous problem sets. The government can impose a transfers T_t on each generation to redistribute resources across generations, such that the period-by-period budget constraint of the generation living in period t reads

$$c_t = w_t + T_t, \tag{1}$$

where c_t denotes the consumption level of each generation. The period-by-period budget constraint of the infinitely-lived government reads

$$b_{t+1} = (1 + r_t)b_t + T_t, \tag{2}$$

where r_t denotes the exogenous interest rate on the international capital market. Without imposing any further restrictions on fiscal policy (except a no-Ponzi condition of course), the net present value budget constraint of the government reads

$$\sum_{t=0}^{\infty} \frac{T_t}{\Pi_{s=0}^{t}(1 + r_s)} = B, \tag{3}$$

such that the present value of all transfers cannot exceed the value of initial assets, B. The government is benevolent towards present and future generations and maximizes a welfare function equal to a weighted sum of each generation's utility

$$\sum_{t=0}^{\infty} \beta_t u(c_t), \quad \beta_0 = 1, \tag{4}$$

where β_t (not to be confused with the discount factor β^t, where t denotes the power of β) denotes the welfare weight that the government puts on generation t.

(a) State the optimality conditions of the government's decision problem (hint: reduce consumption from the problem before maximizing)

$$W_t = \max_{\{c_t, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t u(c_t) \text{ s.t. } (1), (3).$$

Why does the Ricardian equivalence proposition not apply to this economy?

(b) Assume that marginal utility is given by $u'(c) = c^{-\theta}$, $\theta > 0$. Derive the government's Euler equation, by combining the optimality conditions of two subsequent generations, t and $t+1$, respectively.

(c) Solve for c_t as a function of c_0 using the government's Euler equation. Then, only for this subquestion, set the parameter $\theta = 1$ and derive the optimal level of consumption c_0 from Equations (1) and (3).

(d) Consider the Norwegian Handlingsregelen which roughly state that fiscal policy is restricted to be

$$-b_{t+1} = B,$$

for all generations t. Or in words, the government is only allowed to take out the returns on the stock of assets, B. What transfer and private consumption pattern does this imply for each generation? What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

(e) Suppose instead that the government follows the rule

$$-b_t / w_t = B / w_0,$$

for each generation t. Or in words, the government wants to keep the stock of assets as a fraction of wages constant. What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

(f) Suppose that interest rates, $r_t = r$, and wage growth, $g_t = g$, are constant across generations. Calculate the relative welfare weight β_{t+1}/β_t under both fiscal policy rules considered in parts (d) and (e). What policy rule puts a higher relative welfare weight on future generations?

Exercise 4.2: Tax Smoothing

(Note: this is a demanding exercise, so here and there I will provide you with the solutions to provide some explanation and to keep the focus on the optimal fiscal policy part. But no worries, you can do it!) Consider a representative infinitely-lived household with the following preferences over private consumption, c_t, labor supply, h_t, and public goods, g_t

$$U = \max \sum_{t=0}^{\infty} \beta^t \left[\log \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right) + \sigma \log(g_t) \right], \tag{5}$$

where $0 < \varphi < \infty$ denotes the Frisch elasticity of labor supply, and $\sigma > 0$ is a parameter that measures the household's relative preference for public over private goods. The household faces an exogenously given sequence wage rates, w_t, and interest rates, $r_t - \delta$, and labor income is taxed at the proportional rate τ_t in each period such that the net present value budget constraint reads (assume there is also a no-Ponzi condition on asset holdings in the background)

$$\sum_{t=0}^{\infty} \frac{c_t}{\Pi_{s=0}^{t}(1+r_s-\delta)} = a_0 + \sum_{t=0}^{\infty} \frac{(1-\tau_t) w_t h_t}{\Pi_{s=0}^{t}(1+r_s-\delta)}, \tag{6}$$

where the left-hand side is the net present value of consumption, and the right-hand side the net present value of the after-tax income net of the initial asset level, a_0.

(a) Write down the household's optimality conditions with respect to consumption, c_t and labor supply, h_t (the public good provision by the government is taken as given), and derive the optimal labor supply.
Solution:

The households maximizes lifetime utility subject to Equation (6), such that the Lagrangian reads

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[\log\left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right) + \sigma \log(g_t) \right] + \lambda \left[a_0 + \sum_{t=0}^{\infty} \frac{(1-\tau_t) w_t h_t}{\Pi_{s=0}^t (1+r_s-\delta)} - \sum_{t=0}^{\infty} \frac{c_t}{\Pi_{s=0}^t (1+r_s-\delta)} \right].$$

The optimality conditions with respect to consumption c_t and labor supply, h_t, are given by

$$0 = \frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1} - \frac{\lambda}{\Pi_{s=0}^t (1+r_s)}, \quad \forall t,$$

$$0 = \frac{\partial \mathcal{L}}{\partial h_t} = -\beta^t \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1} h_t^{1/\varphi} + \frac{\lambda}{\Pi_{s=0}^t (1+r_s)} (1-\tau_t) w_t, \quad \forall t.$$

Combining the two optimality conditions and substitute for λ to yield

$$\beta \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1} h_t^{1/\varphi} = \beta \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1} (1-\tau_t) w_t,$$

which can be simplified to find the optimal labor supply

$$h_t(\tau_t) = \left[(1-\tau_t) w_t \right]^{\varphi}. \tag{7}$$

(b) Compute the elasticity of the labor supply with respect to the tax rate

$$e(\tau_t) \equiv -\frac{\partial h_t(\tau_t)}{\partial \tau_t} \frac{\tau_t}{h_t(\tau_t)}.$$

(c) Derive the government's labor income tax revenue in each period t as a function of the tax rate, τ_t, - the so called Laffer curve. What tax rate $\bar{\tau}$ is associated with the top of the Laffer curve (the maximum tax revenue), and would a benevolent government ever choose a tax rate above $\bar{\tau}$? What is the tax rate at the top of the Laffer curve if the labor supply is completely inelastic, $\varphi \to 0$, or inelastic, $\varphi \to \infty$?

From now on we assume that $\delta = 1$. Moreover, suppose that the government of this closed economy taxes the total resource income (instead of labor income only, note that this implies that the tax rate that maximizes tax revenue will be somewhat higher than the $\bar{\tau}$ we calculated previously)

$$y_t \equiv w_t h_t + r_t a_t,$$

at the common proportional rate τ_t. Let labor income be a constant fraction $(1-\alpha)$ of the total resource income, $w_t h_t = (1-\alpha)y_t$, and similarly, $r_t a_t = \alpha y_t$, with $0 < \alpha < 1$ (remember that this could be rationalized with a Cobb-Douglas production technology). The household's period-by-period private budget constraint then reads

$$\begin{aligned} c_t + a_{t+1} &= (1-\tau_t)\left[w_t h_t + r_t a_t\right] + (1-\delta)a_t, \quad \delta = 1 \\ &= (1-\tau_t)\left[w_t h_t + r_t a_t\right] = (1-\tau_t)y_t, \quad \forall t, \end{aligned}$$

and the lifetime constraint becomes

$$\sum_{t=0}^{\infty} \frac{c_t}{\Pi_{s=0}^{t}(1-\tau_s)r_s} = a_0 + \sum_{t=0}^{\infty} \frac{(1-\tau_t)w_t h_t}{\Pi_{s=0}^{t}(1-\tau_s)r_s}.$$

Note that the optimal labor supply is not affected by this change of the tax system, but the agent's effective gross interest rate used for discounting is now $1 + (1-\tau_t)r_t - \delta$ and was $(1 - r_t - \delta)$ before.

(d) Derive the household's Euler equation, guess that optimal consumption is proportional to the available resources after taxation,

$$c_t = \mu(1-\tau_t)y_t,$$

show that the Euler equation then implies that $a_{t+1} = \alpha\beta(1-\tau_t)y_t$, and compute the proportional factor μ from the period-by-period budget constraint. (Note: the take-home message from this part will be that the optimal household behavior conditional on the sequence of taxes can be summarized by

$$\begin{aligned} c_t &= (1-\alpha\beta)(1-\tau_t)y_t, \\ h_t &= \left[(1-\tau_t)w_t\right]^{\varphi}, \\ a_{t+1} &= \alpha\beta(1-\tau_t)y_t, \end{aligned}$$

where the available resources y_t can be expressed in terms of labor income

$$y_t = (1-\alpha)^{-1} w_t h_t = (1-\alpha)^{-1} w_t \left[(1-\tau_t)w_t\right]^{\varphi}.$$

Thus, the optimal tax policy will be chosen by the government to maximize the indirect lifetime utility of the household given the above behavior of the agent).

Solution:

The optimality conditions are very similar to before, the only difference is that the after tax gross interest rate used for discounting is $(1-\tau_t)r_t$,

$$0 = \frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1} - \frac{\lambda}{\Pi_{s=0}^t (1-\tau_s) r_s}, \quad \forall t, \tag{8}$$

$$0 = \frac{\partial \mathcal{L}}{\partial h_t} = -\beta^t \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1} h_t^{1/\varphi} + \frac{\lambda}{\Pi_{s=0}^t (1-\tau_s) r_s} (1-\tau_t) w_t, \quad \forall t.$$

To derive the Euler equation divide Equation (8) in period t by the one in period $t+1$

$$\frac{\lambda / \Pi_{s=0}^t (1-\tau_s) r_s}{\lambda / \Pi_{s=0}^{t+1} (1-\tau_s) r_s} = \frac{\beta^t \left(c_t - \frac{h_t^{1+1/\varphi}}{1+1/\varphi} \right)^{-1}}{\beta^{t+1} \left(c_{t+1} - \frac{h_{t+1}^{1+1/\varphi}}{1+1/\varphi} \right)^{-1}},$$

which can be written as

$$\beta(1-\tau_{t+1}) r_{t+1} = \beta(1-\tau_{t+1}) \alpha \frac{y_{t+1}}{a_{t+1}} = \frac{c_{t+1} - \frac{1}{1+1/\varphi} h_{t+1}^{1+1/\varphi}}{c_t - \frac{1}{1+1/\varphi} h_t^{1+1/\varphi}}.$$

Use the guess on the form of optimal consumption, $c_t = \mu(1-\tau_t)w_t$, and also the optimal labor supply, $h_t = [(1-\tau_t)w_t]^\varphi$, to yield

$$\begin{aligned}
\beta(1-\tau_{t+1}) \alpha \frac{y_{t+1}}{a_{t+1}} &= \frac{\mu(1-\tau_{t+1}) y_{t+1} - \frac{1}{1+1/\varphi} \left[(1-\tau_{t+1}) w_{t+1}\right]^{1+\varphi}}{\mu(1-\tau_t) y_t - \frac{1}{1+1/\varphi} \left[(1-\tau_t) w_t\right]^{1+\varphi}} \\
&= \frac{\mu(1-\tau_{t+1}) y_{t+1} - \frac{1}{1+1/\varphi} (1-\tau_{t+1}) w_{t+1} h_{t+1}}{\mu(1-\tau_t) y_t - \frac{1}{1+1/\varphi} (1-\tau_t) w_t h_t} \\
&= \frac{\left(\mu - \frac{1-\alpha}{1+1/\varphi}\right) (1-\tau_{t+1}) y_{t+1}}{\left(\mu - \frac{1-\alpha}{1+1/\varphi}\right) (1-\tau_t) y_t},
\end{aligned}$$

such that indeed

$$a_{t+1} = \alpha\beta(1-\tau_t) y_t.$$

Finally, use the period-by-period constraint to verify that consumption is linear in y_t

$$c_t = (1-\tau_t) y_t - a_{t+1} = (1-\alpha\beta)(1-\tau_t) y_t,$$

such that $\mu = 1-\alpha\beta$.

(e) Compute the indirect utility function, U, of the household as a function of government policy, (τ_t, g_t), wages, w_t, and parameters, $(\alpha, \beta, \varphi, \sigma)$, only.
Solution:

Write consumption as ($y_t = (1-\alpha)^{-1}w_t h_t$)

$$c_t = (1-\alpha\beta)\,(1-\tau_t)y_t = \frac{1-\alpha\beta}{1-\alpha}(1-\tau_t)w_t h_t = \frac{1-\alpha\beta}{1-\alpha}\left[(1-\tau_t)w_t\right]^{1+\varphi}.$$

The indirect utility is then given by

$$\begin{aligned} U &= \sum_{t=0}^{\infty}\beta^t\left[\log\left(\frac{1-\alpha\beta}{1-\alpha}\left[(1-\tau_t)w_t\right]^{1+\varphi} - \frac{1}{1+1/\varphi}\left[(1-\tau_t)w_t\right]^{1+\varphi}\right) + \sigma\log(g_t)\right] \\ &= \sum_{t=0}^{\infty}\beta^t\left[\log\left(\frac{1-\alpha\beta}{1-\alpha} - \frac{1}{1+1/\varphi}\right) + (1+\varphi)\log\left((1-\tau_t)w_t\right) + \sigma\log(g_t)\right]. \end{aligned}$$

(f) The government of this economy chooses fiscal policy to maximize the indirect utility of the household, U, subject to the period-by-period government budget constraint

$$g_t = \tau_t y_t = \tau_t w_t\left[(1-\tau_t)w_t\right]^{\varphi} + \tau_t r_t a_t, \quad \forall t,$$

which implies that the government is restricted to balance the budget (cannot issue government debt) in every period and starts with a zero initial debt position, $b_0 = 0$. State the optimality conditions with respect to public good provision, g_t, and the tax rate, τ_t, and show that the optimal tax rate is the same in each period, $\tau_t = \tau$ (hint: assume for simplicity that τ_t only distorts the labor supply, h_t, and not the savings decision, $\partial a_{t+1}/\partial\tau_t = 0$).

Solution:

The government's Lagrangian is

$$\begin{aligned} \mathcal{L} = \sum_{t=0}^{\infty}\beta^t & \left[\log\left(\frac{1-\alpha\beta}{1-\alpha} - \frac{1}{1+1/\varphi}\right) + (1+\varphi)\log\left((1-\tau_t)w_t\right) + \sigma\log(g_t)\right] \\ & + \gamma_t\left[\tau_t w_t\left[(1-\tau_t)w_t\right]^{\varphi} + \tau_t r_t a_t - g_t\right]. \end{aligned}$$

The optimality conditions for public good provision, g_t, and the tax rate, τ_t, are given by

$$0 = \frac{\partial\mathcal{L}}{\partial g_t} = \frac{\beta^t\sigma}{g_t} - \gamma_t, \quad \forall t, \tag{9}$$

$$0 = \frac{\partial\mathcal{L}}{\partial\tau_t} = \frac{\beta^t(1+\varphi)}{(1-\tau_t)w_t}(-w_t) - \gamma_t\left[w_t\left[(1-\tau_t)w_t\right]^{\varphi}\left[1-e(\tau_t)\right] + r_t a_t\right], \quad \forall t, \tag{10}$$

$$0 = \frac{\partial\mathcal{L}}{\partial\gamma_t} = \tau_t y_t - g_t, \quad \forall t, \tag{11}$$

where γ_t denotes the Lagrange multiplier on the government's period-by-period budget constraint. Note that the form of Equation (10) is the same as in the first-order condition that characterizes the top of the Laffer curve. Combine Equation (9) with Equation (10) to eliminate the Lagrange multiplier yielding

$$\frac{1+\varphi}{(1-\tau_t)} = \frac{\sigma}{g_t}\left[w_t\left[(1-\tau_t)w_t\right]^{\varphi}\left[1-e(\tau_t)\right] + r_t a_t\right],$$

which can be solved for the government expenditure level

$$\begin{aligned} g_t &= \frac{\sigma}{1+\varphi}\left[(1-\tau_t)w_t\left[(1-\tau_t)w_t\right]^{\varphi}\left[1-e(\tau_t)\right]+(1-\tau_t)r_t a_t\right] \\ &= \frac{\sigma}{1+\varphi}\left[(1-\alpha)\left[1-e(\tau_t)\right]+\alpha\right](1-\tau_t)y_t. \end{aligned}$$

Remember that the period-by-period budget constraint implies restricts government expenditures to $g_t = \tau_t y_t$ such that the optimal tax rate has to satisfy

$$g_t = \tau_t y_t = \frac{\sigma}{1+\varphi}\left[(1-\alpha)\left[1-e(\tau_t)\right]+\alpha\right](1-\tau_t)y_t$$

which can be written as (divide by $(1-\tau_t)y_t$ and multiply by φ)

$$e(\tau_t) = \varphi\frac{\tau_t}{1-\tau_t} = \frac{\sigma\varphi}{1+\varphi}\left[1-(1-\alpha)e(\tau_t)\right],$$

such that the elasticity at the optimal tax rate is (note that $(1-\alpha)^{-1}$ is the top of the Laffer curve here, when $\sigma \to \infty$ you would go there)

$$e(\tau_t) = \left[\frac{1+\varphi}{\sigma\varphi}+(1-\alpha)\right]^{-1} < (1-\alpha)^{-1}$$

such that the optimal tax rate is given by

$$\tau_t = \frac{\left[\frac{1+\varphi}{\sigma\varphi}+(1-\alpha)\right]^{-1}}{\varphi+\left[\frac{1+\varphi}{\sigma\varphi}+(1-\alpha)\right]^{-1}} = \frac{1}{1+\left[\frac{1+\varphi}{\sigma}+\varphi(1-\alpha)\right]} < 1.$$

This implies that the optimal tax rate is the same in each period, $\tau_t = \tau$, as the tax elasticity of the labor supply has to be the same in each period. Or in other words, the government is perfectly smoothing taxes across time to minimize the cumulative tax distortion.