Final Exam
ECON 4310, Fall 2015

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Grade: __________

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

XX Instruction for graders:
In general short questions are rewared with either full (10) or no (0) points. Full points are allocated if the correct short answer True/False is provided AND accompanied by a correct explanation. On exception, if someone forgot to give the short answer, but provided an explanation that clearly indicates the right short question, or if some other borderline case occurs, then it is upon your judgement to allocate half (5) of the points. But this should not be the rule but the exception. Be strict in grading those questions, each of them has been discussed extensively in the seminars or in the lecture. XX

Exercise A.1: (10 Points) The Intertemporal Elasticity of Substitution

Consider the optimal intertemporal consumption choice of a household in discrete and infinite time. The representative household has preferences $U = \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0,1)$ is the discount factor and the momentary utility function is

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta}, \quad \theta > 1.$$

The optimal behavior is characterized by the following consumption Euler equation

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{1+r_{t+1}}.$$

The Intertemporal Elasticity of Substitution (IES)

$$\text{EIS} \equiv \frac{\partial \log(c_{t+1}/c_t)}{\partial \log(1+r_{t+1})},$$

does *not* depend on the discount factor β for this momentary utility function. True or false?

Your Answer:

True: ☒ False: ☐

The EIS for this class of preferences is constant and only depends on the parameter θ (see calculation below). First, the marginal utility implied by the functional form of the utility function is

$$u'(c) = c^{-\theta},$$

and the consumption Euler equation then reads

$$\beta \left(\frac{c_{t+1}}{c_t} \right)^{-\theta} = (1 + r_{t+1})^{-1},$$

Take logs on both sides of the equation

$$\log \beta - \theta \log \left(\frac{c_{t+1}}{c_t} \right) = -\log(1 + r_{t+1}),$$

to yield

$$\text{EIS} = \frac{\partial \log(c_{t+1}/c_t)}{\partial \log(1 + r_{t+1})} = 1/\theta.$$

Exercise A.2: (10 Points) A Solow Model without productivity growth

Consider the following version of the Solow model, with an alternative production function specified in (2), a growing population, L_t, and a constant saving rate, s, as described by the following equations

$$K_{t+1} = sY_t + (1-\delta)K_t \tag{1}$$
$$Y_t = \left[K_t^{\rho} + L_t^{\rho}\right]^{1/\rho}, \quad \rho > 0, \tag{2}$$
$$L_{t+1} = (1+n)L_t,\ L_0 > 0, A = 1,$$

where $0 < \delta < 1$ is the depreciation rate of physical capital, and $n + \delta > s$. (Hint: you will have to repeatedly use the fact that

$$x_t^{-1}\left[K_t^{\rho} + L_t^{\rho}\right]^{1/\rho} = \left[K_t^{\rho}x_t^{-\rho} + L_t^{\rho}x_t^{-\rho}\right]^{1/\rho},$$

in your explanation.)

The steady-state level of the capital stock per capita, $k_t \equiv K_t/L_t$, is given by

$$k^{\star} = \left[\left(\frac{n+\delta}{s}\right)^{\rho} - 1\right]^{-1/\rho}.$$

True or false?

Your Answer:

True: ☒ False: ☐

First, detrend the output to yield

$$y_t = \frac{Y_t}{L_t} = \left[K_t^{\rho}L_t^{-\rho} + L_t^{\rho}L_t^{-\rho}\right]^{-\rho}$$
$$= \left[k_t^{\rho} + 1\right]^{1/\rho}.$$

Then detrend both sides of Equation (1) by multiplying by $1/L_t$

$$\frac{K_{t+1}}{L_{t+1}}\frac{L_{t+1}}{L_t} = sy_t + (1-\delta)k_t \quad \Leftrightarrow \quad k_{t+1}(1+n) = s\left[k_t^{\rho} + 1\right]^{1/\rho} + (1-\delta)k_t.$$

Then impose the steady-state condition, $k_{t+1} = k_t = k^{\star}$, on this equation to yield

$$k^{\star}(1+n) = s\left[(k^{\star})^{\rho} + 1\right]^{1/\rho} + (1-\delta)k^{\star} \quad \Leftrightarrow \quad k^{\star}(n+\delta) = s\left[(k^{\star})^{\rho} + 1\right]^{1/\rho}.$$

Finally, divide both sides by $k^{\star}$ and s

$$\frac{n+\delta}{s} = \left[(k^{\star})^{\rho}(k^{\star})^{-\rho} + (k^{\star})^{-\rho}\right]^{1/\rho},$$

such that

$$k^{\star} = \left[\left(\frac{n+\delta}{s}\right)^{\rho} - 1\right]^{-1/\rho}.$$

Exercise A.3: (10 Points) War spending in the Ramsey model

Consider the dynamic equilibrium equations of the Ramsey model

$$\frac{c_{t+1}}{c_t} = \left[\beta(1+\alpha k_{t+1}^{\alpha-1} - \delta)\right]^{1/\theta}, \quad c_t \equiv C_t/(A_t L_t),$$
$$k_{t+1} - k_t = k_t^{\alpha} - \delta k_t - c_t - \tau, \quad k_t \equiv K_t/(A_t L_t),$$

where K_t is the aggregate capital stock, $\alpha k_{t+1}^{\alpha-1} - \delta$ the interest rate, A_t is the state of technology, L_t the growing size of the population, C_t aggregate consumption, $\alpha \in (0,1)$ the capital income share in the economy, $\delta \in (0,1)$ the depreciation rate of physical capital, $\beta \in (0,1)$ is the subjective discount factor, and $1/\theta$ the intertemporal elasticity of substitution. Finally, τ is the lump-sum tax which finances the constant government expenditure according to a balanced budget,

$$\tau = G.$$

Suppose that the economy is in the steady-state. Consider now a temporary and unexpected increase of the lump-sum tax, τ, to finance an unexpected war.

As a consequence, consumption per efficiency unit, c_t, will jump upwards on impact and then fall in the transition back to the steady-state. True or false?

Your Answer:

True: ☐ False: ☒

Consumption per efficiency unit, c_t, must jump downwards on impact, because the household has less after-tax resources availabe to spend on private consumption and investment. Graphically, the shock will lead to a temporary shift of the $\dot{k} = 0$ locus downwards such that consumption jumps downwards on impact at the given steady-state capital stock. During and after the shock, consumption will gradually increase and converge back to the initial steady-state level.

Exercise A.4: (10 Points) Income, substitution and wealth effect

Consider a representative consumer who lives for only two periods denoted by $t = 1, 2$. The consumer is born in period 1 without any financial assets and leaves no bequests or debt at the end of period 2. The consumer's labor income is w_t in each period and her preferences over consumption can be represented by the utility function

$$U(c_1, c_2) = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1, \tag{3}$$

where the momentary utility function is given by

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta}, & \theta \neq 1. \\ \log(c), & \theta = 1, \end{cases}$$

with $\theta > 0$. The optimal consumption in period 1 is given by

$$c_1 = \frac{1}{1 + \beta^{1/\theta}(1+r)^{1/\theta-1}} \left(w_1 + \frac{w_2}{1+r} \right).$$

Now, consider the effect of an increase in the gross real interest rate $1 + r$ on the optimal first period consumption. The overall effect of this increase is always strictly positive if the labor income in the second period is zero, $w_2 = 0$. True or false?

Your Answer:

True: ☐ False: ☒

If $w_2 = 0$ (no wealth effect), the overall effect is positive only if $\theta > 1$, that is the income effect dominates the substitution effect. The derivative with respect to $1 + r$ reads

$$\begin{aligned} \partial c_1 / \partial(1+r) &= -\frac{(1/\theta - 1)\beta^{1/\theta}(1+r)^{1/\theta-2}}{\left[1 + \beta^{1/\theta}(1+r)^{1/\theta-1}\right]^2} \left(w_1 + \frac{w_2}{1+r} \right) \\ &\quad - \frac{1}{1 + \beta^{1/\theta}(1+r)^{1/\theta-1}} \frac{w_2}{(1+r)^2} \\ &= -\frac{(1/\theta - 1)\beta^{1/\theta}(1+r)^{1/\theta-2}}{\left[1 + \beta^{1/\theta}(1+r)^{1/\theta-1}\right]^2} w_1, \quad w_2 = 0. \end{aligned}$$

Exercise A.5: (10 Points) Optimal policy, Laffer curve

Suppose the aggregate labor supply, $h(\tau)$, of an economy as a function of the labor income tax rate, τ, is given by

$$h(\tau) = [(1-\tau)w]^{\varphi}.$$

where φ is the Frisch elasticity of labor supply. Suppose that the government must raise a level of tax revenue to finance an expenditure $g^{\star}$, where

$$0 < g^{\star} < \bar{\tau} w h(\bar{\tau}),$$

and $\bar{\tau}$ is the tax rate associated with the top of the Laffer curve.

The optimal labor income tax rate to finance $g^{\star}$ is only one, and it is $\tau = \bar{\tau}$.
True or false?

Your Answer:

True: ☐ False: ☒

The optimal tax rate is only one, but (unless $g^{\star} = \bar{\tau} w h(\bar{\tau})$) it will always be strictly smaller than $\bar{\tau}$. A tax rate of $\bar{\tau}$ would raise excessive tax revenue and create unnecessary labor supply distortions.

Exercise A.6: (10 Points) Optimal policy: fully-funded pension system

Consider the simple two period life-cycle overlapping generations model. The representative agent lives for two period $t = 1, 2$, and has a labor income $w_1 > 0$ in the first period, no labor income in the second period, $w_2 = 0$, and can save across periods. The agent's preferences can be represented by the utility function

$$U(c_1, c_2) = \log(c_1) + \beta \log(c_2), \quad 0 < \beta < 1,$$

and she is subject to the lifetime budget constraint

$$c_1 + \frac{c_2}{1+r} = w_1,$$

where $r > 0$ is a given exogenous interest rate. The government considers the introduction of a compulsory fully-funded pension system, such that the agent is obliged to contribute a lump-sum $\tau > 0$ in the first period, and receives a pension

$$P = (1 + \tilde{r})\tau,$$

in the second period. Moreover, suppose the government has better investment opportunities than the agent such that $\tilde{r} > r$.

The introduction of the pension scheme will never affect the optimal consumption choice of the agent. True or false?

Your Answer:

True ☐ False: ☒

After the introduction of the pensions system the lifetime budget constraint of the agent reads

$$c_1 - \tau + \frac{c_2}{1+r} = w_1 + \frac{P}{1+r} \quad \Leftrightarrow \quad c_1 + \frac{c_2}{1+r} = w_1 + \frac{\tilde{r} - r}{1+r}\tau > w_1,$$

which shows that the lifetime income of the agent increases with the fully-funded pension system. Because the agent's marginal utility of consumption is always strictly positive, she will optimally increase lifetime consumption after the introduction of the pension system to fully exhaust the lifetime budget.

Exercise B:
Long Question (60 Points)

A comparison of the Solow model and the Ramsey growth model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function

$$Y_t = F(K_t, L) = K_t^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is aggregate capital and L is the number of workers in the economy. The law of motion for aggregate capital is given by

$$K_{t+1} = (1-\delta) K_t + I_t, \quad K_0 > 0, \tag{4}$$

where I_t denotes aggregate investment, and $0 < \delta < 1$ the depreciation rate. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that consumption per worker, c_t, is the same as aggregate consumption, $C_t = c_t = c_t L$.

Consider now two different models. The Solow model where agents have a constant savings rate, s, such that

$$I_t = sY_t.$$

And the Ramsey growth model where household utility is maximized

$$\sum_{t=0}^{\infty} \beta^t u(C_t), \tag{5}$$

such that aggregate investment (savings) is endogenous

$$I_t = Y_t - C_t.$$

In both models markets are competitive, thus input factors K_t and L are paid their marginal products.

(a) (10 Points) Compute the interest rate in this Solow model's stable steady state. (hint: compute the stable aggregate steady-state capital stock first.)
Solution:

XX Allocation of points:
(i) derivation of the interest as a function of the capital stock (3 points, PLEASE DEDUCT 1 POINT if the δ gets forgotten here, but give full points in any future expression where the δ still does not show up), (ii) derivation of the steady-state capital stock, (5 points), (iii) derivation of the steady-state interest (combination of (i) and (ii) such that the interest rate is a function of parameters only), (2 points). PLEASE CHECK whether the interest rate is derived in later parts, probably in (b), (c), or (d), and allocate points here accordingly. XX

Input factors are priced according to the marginal product, thus the interest rate in any given period is given by

$$r_t - \delta = \frac{\partial Y_t}{\partial K_t} - \delta = \alpha K_t^{\alpha-1} L^{1-\alpha} - \delta = \alpha K_t^{\alpha-1} - \delta.$$

As there is no exogenous growth in technology or population, aggregate capital will be constant such that the steady-state condition reads

$$K^\star = (1-\delta)K^\star + s(K^\star)^\alpha.$$

Because we are looking for the stable steady-state, $K^\star > 0$, we can divide by $K^\star$ to yield

$$\delta(K^\star)^{1-\alpha} = s \quad \Leftrightarrow \quad K^\star = \left(\frac{s}{\delta}\right)^{1/(1-\alpha)}.$$

Thus, the interest rate in the stable steady-state reads

$$r^\star - \delta = \alpha(K^\star)^{\alpha-1} - \delta = \alpha\left(\frac{s}{\delta}\right)^{\frac{\alpha-1}{1-\alpha}} - \delta = \frac{\alpha\delta}{s} - \delta.$$

(b) (10 Points) In the Ramsey growth model households maximize lifetime utility in Equation (5) with respect to C_t and K_{t+1} and subject to the law of motion of capital stated in Equation in (4). Taking into account the functional form of output, Y_t, and investment, I_t, and that $L = 1$, write up the Lagrangian of this maximization problem and derive the following optimality conditions of the Ramsey growth model

$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta\left[1 + \alpha(K_{t+1})^{\alpha-1} - \delta\right]$$
$$K_{t+1} - K_t = K_t^\alpha - \delta K_t - C_t.$$

The first optimality condition is the model's Euler equation and the second the resource constraint.

Solution:

XX Allocation of points:
(i) Lagrangian, (4 points), (ii) optimality conditions in raw form, (3 points), (iii) derivation of the Euler equation, (3 points). XX

The Lagrangian reads

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u\left(C_t\right) + \lambda_t \left[K_t^\alpha + (1-\delta) K_t - C_t - K_{t+1}\right],$$

with associated optimality conditions

$$0 = \frac{\partial \mathcal{L}}{\partial C_t} = \beta^t u'(C_t) - \lambda_t$$
$$0 = \frac{\partial \mathcal{L}}{\partial K_{t+1}} = -\lambda_t + \lambda_{t+1}\left[\alpha K_{t+1}^{\alpha-1} + (1-\delta)\right]$$
$$0 = \frac{\partial \mathcal{L}}{\partial \lambda_t} = K_t^{\alpha} + (1-\delta) K_t - C_t - K_{t+1}.$$

Eliminating the Lagrange multiplier the second optimality condition can be written as

$$u'(C_t) = \beta u'(C_{t+1})\left[\alpha K_{t+1}^{\alpha-1} + (1-\delta)\right]$$

Thus the optimality conditions can be summarized as

$$\frac{u'(C_t)}{u'(C_{t+1})} = \beta\left[1 + \alpha (K_{t+1})^{\alpha-1} - \delta\right]$$
$$K_{t+1} - K_t = K_t^{\alpha} - \delta K_t - C_t.$$

(c) (5 Points) Compute the interest rate in the Ramsey model in a steady-state. (hint: you can solve this part even if you were not able to solve part (b).)
Solution:

XX Allocation of points:
(i) stating steady-state version of the Euler equation, (2 points), (ii) derivation of the steady-state interest rate as a function of parameters only), (3 points). XX

Since there is no exogenous growth, aggregate consumption will be constant in a steady-state. The Euler equation then implies that

$$1 = \beta\left[\alpha (K^{\star})^{\alpha-1} + (1-\delta)\right] \quad \Leftrightarrow \quad 1 = \beta\left[1 + r^{\star} - \delta\right],$$

such that the steady-state interes rate is given by

$$r^{\star} - \delta = 1/\beta - 1.$$

(d) (10 Points) Does the interest rate in the Solow model depend on the saving rate s in the stable steady-state? Is the intereste rate increasing or decreasing in s or independent from s. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)
Solution:

Yes, the interest rate in the steady-state decreases in the savings rate (*XX (right answer, 5 points) XX*). The reason is that the aggregate steady state capital stock is increasing in the savings rate because with higher savings a higher amount of depreciating capital can be offset to hold the aggregate capital stock constant. With the production function under consideration, the marginal product of capital decreases with the amount of capital, thus a higher steady-state capital stock will be reflected in a lower interest rate (*XX (correct economic reasoning, 5 points, if reasoning is rather technical and not very deep then allocate 2 out of 5 points) XX*).

(e) (10 Points) Does the interest rate in the Ramsey model depend on the discount factor β in a steady-state? Is the interest rate increasing or decreasing in β or independent from β. Why? (hint: you can try to answer this question even though you were not able solve previous parts.)
Solution:

Yes, the interest in the steady-state decreases in the discount factor β (*XX (right answer, 5 points) XX*). The reason is that the aggregate steady state capital stock is increasing in the discount factor as more patient households will want to shift resources into the future by saving more (in capital). With the production function under consideration, the marginal product of capital decreases with the amount of capital, thus a higher steady-state capital stock will be reflected in a lower interest rate (*XX (correct economics reasoning, 5 points, if reasoning is rather technical and not very deep then allocate 2 out of 5 points) XX*).

(f) (5 Points) Compute the saving rate $\bar{s}$ which gives the same steady-state capital stock in the Solow model as in the Ramsey growth model. Is this saving rate, $\bar{s}$, increasing or decreasing in the discount factor β?
Solution:

In the Ramsey model, the steady-state capital stock is characterized by the Euler equation

$$1 = \beta \left[\alpha (K^\star)^{\alpha-1} + (1-\delta) \right] \quad \Leftrightarrow \quad K^\star = \left[\frac{\alpha}{1/\beta - (1-\delta)} \right]^{1/(1-\alpha)}.$$

Thus, the saving rate in the the Solow model that yields the same steady-state capital stock as in the Ramsey model is characterized by (*XX (right calculation, 3 points) XX*)

$$\left(\frac{\bar{s}}{\delta} \right)^{1/(1-\alpha)} = \left[\frac{\alpha}{1/\beta - (1-\delta)} \right]^{1/(1-\alpha)} \quad \Leftrightarrow \quad \bar{s} = \frac{\delta\alpha}{1/\beta - (1-\delta)}.$$

According to the previous analysis (and also if you do the math), as the steady-state capital stock is increasing in s in the Solow model and in β in the Ramsey growth model, $\bar{s}$ must be increasing in β (*XX (right answer, 2 points) XX*).

(g) (10 Points) Compute the the saving rate $\hat{s}$ which gives the same interest rate in the Solow model as in the Ramsey growth model. Is this saving rate increasing or decreasing in the depreciation rate δ?

Solution:

The savings rate is the same as before, $\hat{s} = \bar{s}$ (*XX (right insight, 5 points) XX*), as the interest rate is a function of the aggregate capital stock and the parameters α and δ only. The derivative of $\hat{s}$ with respect to δ reads

$$\begin{aligned}\frac{\partial \hat{s}}{\partial \delta} &= \frac{\alpha}{1/\beta - (1-\delta)} - \frac{\alpha\delta}{[1/\beta - (1-\delta)]^2} \\ &= \frac{\alpha/\beta - (1-\delta)\alpha - \alpha\delta}{[1/\beta - (1-\delta)]^2} \\ &= \frac{\alpha(1/\beta - 1)}{[1/\beta - (1-\delta)]^2} > 0,\end{aligned}$$

such that this savings rate is increasing in the depreciation rate δ. (*XX (right answer and calculation, 5 points) XX*).

Exercise C:
Long Question (60 Points)

Consumption response to income shocks

Consider a household decision problem under uncertainty, when preferences are linear-quadratic:

$$u(c_t) = c_t - \frac{b}{2}c_t^2, \quad b \geq 0.$$

The household lives from period 0 to period $T < +\infty$ and discounts the future at rate $\beta = 1$. Assume $1 + r = 1/\beta = 1$, that is $r = 0$. The lifetime budget constraint of the household viewed from period 0 reads

$$(1+r)\,a_0 + \sum_{t=0}^{T} \frac{I_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{c_t}{(1+r)^t}, \tag{6}$$

and viewed from period 1

$$(1+r)\,a_1 + \sum_{t=1}^{T} \frac{I_t}{(1+r)^t} = \sum_{t=1}^{T} \frac{c_t}{(1+r)^t}, \tag{7}$$

where I_t is income in period t which is uncertain in any previous period $s \leq t$ and only learned in period t. Expected income (or, consumption) in period t viewed from period $s \leq t$ equals $E_s[I_t]$ (or, $E_s[c_t]$), and the Euler equation is given by

$$u'(c_t) = \beta(1+r)E_t\left[u'(c_{t+1})\right], \quad \forall t \leq T-1. \tag{8}$$

(a) (10 Points) Under the given assumptions, (i) show that the Euler equation can be written as

$$c_t = E_t[c_{t+1}], \quad \forall t \leq T-1, \tag{9}$$

and, (ii) give an interpretation (just one sentence!) of Equation (9).

Solution:

XX Allocation of points:
(i) derivation of the Euler equation (5 points), (ii) correct interpretation of Equation (9) (5 points). XX

(i) Since $\beta(1+r) = 1$, and marginal utility is given by

$$u'(c_t) = 1 - bc_t,$$

the Euler equation (8) can be written as

$$1 - bc_t = \beta(1+r)E_t[1 - bc_{t+1}] \quad \Leftrightarrow \quad c_t = E_t[c_{t+1}].$$

(ii) The above Euler equation implies that future consumption is expected to be the same as today's consumption. *XX The following step is just for illustration, and*

NOT required in the answer to the question. XX The law of iterated expectations then implies

$$\begin{aligned} E_s[c_t] &= E_s[E_{s+1}[\ldots E_{t-2}[E_{t-1}[c_t]]\ldots]] \\ &= E_s[E_{s+1}[\ldots E_{t-2}[c_{t-1}]\ldots]] \\ &= E_s[E_{s+1}[c_{s+2}]] \\ &= E_s[c_{s+1}] = c_s. \end{aligned}$$

(b) (10 Points) Note that Equation (9) together with the law of iterated expectations implies that

$$E_s[c_t] = c_s, \quad \forall s \leq t.$$

Now, assume that the household expects to not pay any taxes such that income I_t is equal to wage income w_t in any period t. Take expectations on the lifetime budget constraints and show that consumption in period 0 and 1 are given by

$$c_0 = (T+1)^{-1}\left(a_0 + \sum_{t=0}^{T} E_0[I_t]\right), \quad E_0[I_t] = E_0[w_t],$$

and

$$c_1 = T^{-1}\left(a_1 + \sum_{t=1}^{T} E_1[I_t]\right), \qquad E_1[I_t] = E_1[w_t],$$

respectively.

Solution:

XX Allocation of points:
(i) taking expectation of the lifetime budget constraint (2 points for each $s = 0, 1$), (ii) solving for consumption (3 points for each $s = 0, 1$). PLEASE, DO NOT AWARD POINTS if consumption is not derived XX

(i) Take expectations from period $s = 0, 1$ on both sides of the lifetime budget constraint using the fact that $E_s[c_t] = c_s$

$$a_s + \sum_{t=s}^{T} E_s[I_t] = \sum_{t=s}^{T} E_s[c_t] = (T+1-s)c_s,$$

where we have also used the fact that $r = 0$. Divide both sides by $T + 1 + s$ to yield

$$c_s = (T+1-s)^{-1}\left(a_s + \sum_{t=s}^{T} E_s[I_t]\right), \quad s = 0, 1.$$

Finally, expectations income in period s are given by $E_s[I_t] = E_s[w_t]$.

(c) (10 Points) Now assume that the household knows (already in period 0) that from period 1 to period T the government levies taxes g, so that income is $I_t = w_t - g$ from period 1 onwards. Derive again consumption in period 0 and 1.

Solution:

XX Allocation of points:
(i) consumption in period 0 (5 points), (ii) consumption in period 1 (5 points). PLEASE ALLOCATE 2 POINTS for (i) and (ii), if the g is correctly taken into account, but the expectations are wrong, i.e., $E_s[I_t]$) instead of $E_s[w_t]$. XX

Simply use the information in the above derived expressions in period 0 consumption

$$c_0 = (T+1)^{-1}\left(a_0 + w_0 + \sum_{t=1}^{T}\left(E_0\left[w_t\right] - g\right)\right)$$
$$= (T+1)^{-1}\left(a_0 + \sum_{t=0}^{T} E_0\left[w_t\right]\right) - \frac{T}{T+1}g,$$

and in period 1 consumption is

$$c_1 = T^{-1}\left(a_1 + \sum_{t=1}^{T}\left(E_1\left[w_t\right] - g\right)\right)$$
$$= T^{-1}\left(a_1 + \sum_{t=1}^{T} E_1\left[w_t\right]\right) - g.$$

(d) (10 Points) Assume now that in period 0 the household expects to never pay taxes. In period 1 the household is surprised since the government levies now taxes g from period 1 to period T, so that income $I_t = w_t - g$ from period 1 to T. Derive consumption in period 0 and 1.

Solution:

XX Allocation of points:
(i) consumption in period 0 (5 points), (ii) consumption in period 1 (5 points). PLEASE ALLOCATE FULL POINTS, if the impact of g is correctly incorporated, but there is a repeated mistake based on wrong calculations in part (b) or (c). XX

Consumption in period 0 is the same as in part (b)

$$c_0 = (T+1)^{-1}\left(a_0 + \sum_{t=0}^{T} E_0\left[w_t\right]\right),$$

while consumption in period 1 is the same as in part (c)

$$\begin{aligned} c_1 &= T^{-1}\left(a_1 + \sum_{t=1}^{T}\left(E_1\left[w_t\right] - g\right)\right) \\ &= T^{-1}\left(a_1 + \sum_{t=1}^{T} E_1\left[w_t\right]\right) - g. \end{aligned}$$

(e) (10 Points) Assume now that in period 0 the household expects to never pay taxes. In period 1 the household is surprised since the government levies now taxes g but only in period 1. From period 2 to period T no taxes are levied.

Solution:

XX Allocation of points:
(i) consumption in period 0 (5 points), (ii) consumption in period 1 (5 points). PLEASE ALLOCATE FULL POINTS, if the impact of g is correctly incorporated, but there is a mistake based on wrong calculations in part (b), (c), or (d). XX

Consumption in period 0 is the same as in part (b)

$$c_0 = (T+1)^{-1}\left(a_0 + \sum_{t=0}^{T} E_0\left[w_t\right]\right),$$

while consumption in period 1 is

$$\begin{aligned} c_1 &= T^{-1}\left(a_1 + E_1[w_1] - g + \sum_{t=2}^{T} E_1\left[w_t\right]\right) \\ &= T^{-1}\left(a_1 + \sum_{t=1}^{T} E_1\left[w_t\right]\right) - T^{-1}g. \end{aligned}$$

(f) (10 Points) Explain your results and the differences in consumption for the different scenarios in parts (c), (d), and (e), relative to part (b).

Solution:

XX Allocation of points:
(i) correct explanation of scenario in (c) (4 points), (ii) correct explanation of scenario in (d) (3 points), (iii) correct explanation of scenario in (e) (3 points). XX

- In part (c), a permanent and expected decrease in income gives a permanent (net present-value equivalent) reduction in both consumption levels, in period 0 and 1.
- In part (d), a permanent and unexpected decrease in income leaves period 0 consumption unchanged, but gives a one-to-one reduction in period 1 consumption.

- In part (e), a temporary and unexpected decrease in income leaves period 0 consumption unchanged, and gives only a small increase in period 1 consumption, because in lifetime value the one-period increase in the tax has less impact than the permanent increase in part (d).
