Exercise 6.2: The Laffer Curve

Consider a representative household of a static economy with the following preferences over private consumption, c, labor supply, h, and public goods, g

$$U = \max \left[\frac{\left(c - \frac{h^{1+1/\varphi}}{1+1/\varphi}\right)^{1-\theta} - 1}{1-\theta} + \sigma \log(g) \right], \quad \theta > 0, \tag{1}$$

where $0 < \varphi < \infty$ denotes the Frisch elasticity of labor supply, and $\sigma > 0$ is a parameter that measures the household's relative preference for public over private goods. The household faces an exogenously given wage rate, w, and interest rate, r, and labor income is taxed at the proportional rate τ^n yielding the private budget constraint (the household is born without assets)

$$c = (1-\tau^n)wh. \tag{2}$$

(a) Write down the household's optimality conditions with respect to consumption, c and labor supply, h (the public good provision by the government is taken as given), and derive the optimal labor supply which we will denote by $h(\tau^n)$.

(b) Compute the elasticity of the labor supply with respect to the tax rate

$$e(\tau^n) \equiv -\frac{\partial h(\tau^n)}{\partial \tau^n} \frac{\tau^n}{h(\tau^n)}.$$

Show that this elasticity is increasing in the tax rate, τ^n, i.e., the higher the tax rate the more distorted is the labor supply in this economy.

(c) Derive the government's labor income tax revenue as a function of the tax rate - the so called Laffer curve. What tax rate $\bar{\tau}$ is associated with the top of the Laffer curve (the maximum tax revenue)? What value takes the elasticity $e(\tau)$ at the top of the Laffer curve? What was the tax rate at the top of the Laffer curve if the labor supply is completely inelastic, $\varphi \to 0$, or inelastic, $\varphi \to \infty$?

Solution:

The tax revenue of the government is given by

$$\tau^n w h(\tau^n) = \tau^n w \left[(1-\tau^n)w\right]^{\varphi},$$

such that the top of the Laffer curve is characterized by

$$\bar{\tau} = \arg \max_{0 \le \tau^n \le 1} \tau^n w \left[(1-\tau^n)w\right]^{\varphi}.$$

The first-order condition reads

$$\begin{aligned} 0 &= w\left[(1-\bar{\tau})w\right]^{\varphi} - \varphi \bar{\tau} w \left[(1-\bar{\tau})w\right]^{\varphi-1} w \\ &= w\left[(1-\bar{\tau})w)\right]^{\varphi} \left[1 - \varphi\bar{\tau}(1-\bar{\tau})^{-1}\right] = w\left[(1-\bar{\tau})w)\right]^{\varphi}\left[1 - e(\bar{\tau})\right]. \end{aligned}$$

The first-order condition suggests $\bar{\tau} = 1$ as one of the candidate solutions. However, since $\bar{\tau} = 1$ implies a tax revenue of zero we can safely drop it as a maximum candidate. Due to the ruling out the unit tax rate, we can divide both sides of the above optimality condition by $w\left[(1-\bar{\tau})w\right]^{\varphi}$ to characterize the tax rate that maximizes tax revenue

$$1 = \varphi\bar{\tau}(1-\bar{\tau})^{-1} = e(\bar{\tau}) \quad \Leftrightarrow \quad \bar{\tau} = \frac{1}{1+\varphi}.$$

The above equation implies that the elasticity at the top of the Laffer curve is exactly 1. The two limit cases yield

$$\lim_{\varphi\to 0} \bar{\tau} = 1, \quad \lim_{\varphi\to\infty} \bar{\tau} = 0,$$

Thus, the less elastic the labor supply is, the higher is the maximum revenue that the government can generate from taxing labor income. Note that the case of lump-sum taxation is nested when $\varphi \to 0$, such that the government generates the maximum tax revenue at the unit tax rate.

Note that $\bar{\tau} = 1/(1+\varphi)$ is indeed a maximizer as the tax revenue is globally concave

$$\begin{aligned}
\frac{\partial[\tau^n wh(\tau)]}{\partial\tau^n} &= w\left[(1-\tau^n)w)\right]^{\varphi}\left[1-e(\tau^n)\right] \\
\frac{\partial^2[\tau^n wh(\tau)]}{\partial\tau^n\partial\tau^n} &= \varphi w\left[(1-\tau^n)w)\right]^{\varphi-1}(-w)\left[1-e(\tau^n)\right] \\
&\quad + w\left[(1-\tau^n)w)\right]^{\varphi}(-1)\frac{\partial e(\tau^n)}{\partial\tau^n} < 0.
\end{aligned}$$

(d) Suppose the government wants to finance the specific level of government expenditure $g^\star$ that is located within the bounds

$$0 < g^\star < \bar{\tau}wh(\bar{\tau}).$$

Assume that $\varphi = 1$. Find the optimal tax rate, $\tau^\star$, to finance the government expenditure level, $g^\star$, with a balanced government budget. Would a benevolent government ever choose a tax rate above $\bar{\tau}$?

Solution:

When $\varphi = 1$ then the top of the Laffere curve is given by $\bar{\tau} = 1/2$ and the maximum tax revenue is

$$\bar{\tau}w\left[(1-\bar{\tau})w\right] = 1/4w^2 > g^\star.$$

The government budget constraint reads

$$\begin{aligned}
g^\star &= \tau^\star wh(\tau^\star) \\
&= \tau^\star w\left[(1-\tau^\star)w\right] = \tau^\star w^2 - (\tau^\star)^2 w^2,
\end{aligned}$$

which can be written as the quadratic equation

$$\begin{aligned} 0 &= w^2(\tau^\star)^2 - w^2\tau^\star + g^\star \\ &\equiv ax^2 + bx + c. \end{aligned}$$

The two solutions are characterized by

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{w^2 \pm \sqrt{w^4 - 4w^2 g^\star}}{2w^2} = 1/2 \pm \frac{\sqrt{1 - 4w^{-2}g^\star}}{2}. \end{aligned}$$

Since it can never be optimal to raise the same tax revenue $g^\star$ with a higher tax rate than necessary (because it would increase the labor supply distortion and reduce the available budget of the household unnecessarily), the optimal tax rate must be given by

$$\tau^\star = 1/2 - \frac{\sqrt{1 - 4w^{-2}g^\star}}{2} < \bar{\tau} = 1/2.$$

Note that the term under the square root is strictly positive since $g^\star < 1/4w^2$.