Problem Set 8
ECON 4310, Fall 2015

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**XX**
Exercise B		**XX**
⋮		⋮
Σ		**180**

Grade: __________

Exercise A:
Short Questions (XX Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to reason your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

(XX Points) Exercise A.1: Precautionary savings

Consider the simple two-period real business cycle model discussed in the seminar and in the lecture. With an asset supply of zero, $w_1 = \mathrm{E}[w(s_2)]$, and an optimal consumption profile, $c_1 = w_1$, $c_2(s_2) = w(s_2)$, the stochastic consumption Euler equation in this model is given by

$$\beta(1+r_2) = \frac{u'(c_1)}{\mathrm{E}[u'(c_2(s_2))]} = \frac{u'(\mathrm{E}[w(s_2)])}{\mathrm{E}[u'(w(s_2))]}.$$

The stochastic process for the wage in the second period, $w(s_2)$, takes the form

$$w(s_2) = \begin{cases} w(s_G) = 1 + \sigma/2, & \text{with prob. } 1/2, \\ w(s_B) = 1 - \sigma/2, & \text{with prob. } 1/2, \end{cases}$$

where $\sigma \in (0,2)$ parametrizes the risk in this economy. Assume that the utility function is of the following form

$$u(c) = c - \frac{1}{4}c^2, \ \forall c \in [0,2].$$

Does this utility function, $u(c)$, imply precautionary savings?

Your Answer:

Yes: ☐ No: ☐

(XX Points) Exercise A.2: Asset pricing

Let m_{t+1} and x_{t+1} be two random scalars. Then

$$E_t[m_{t+1}x_{t+1}] = E_t[m_{t+1}]E_t[x_{t+1}] + 2Cov_t[m_{t+1}, x_{t+1}],$$

where

$$Cov_t(m_{t+1}, x_{t+1}) \equiv E_t\left[(m_{t+1} - E_t[m_{t+1}])\,(x_{t+1} - E_t[x_{t+1}])\right].$$

Your Answer:

True: ☐ False: ☐

(XX Points) Exercise A.X: Some more short questions follow ...

Exercise B:
Long Question (XX Points)

Consumption-based asset pricing

Consider a representative household of an economy that maximizes expected lifetime utility

$$U(c_t, c_{t+1}(s_{t+1})) = \frac{c_t^{1-\theta}}{1-\theta} + \beta E_t\left[\frac{c_{t+1}(s_{t+1})^{1-\theta}}{1-\theta}\right], \quad 0 < \beta < 1,$$

by choosing consumption c_t and c_{t+1} in both periods. The agent is endowed with the income e_t and e_{t+1} in the corresponding periods (let e_{t+1} be stochastic), and can shift resources across periods by borrowing or lending in a risk free bond, b_{t+1}, and in a risky asset, a_{t+1}. The risk free bond pays 1 unit of consumption for sure in the future period and has a price q_t in terms of today's consumption, while the risky asset's payoff is the realization of a random variable $x_{t+1} \equiv p_{t+1} + d_{t+1}$ and is of price p_t in terms of today's consumption. The agent initially owns neither assets nor bonds. Expectations are with respect to the future state, $s_{t+1} = (e_{t+1}, x_{t+1}) \in S$, where the set S contains the possible realizations of the state, s_{t+1}.

(a) (XX Points) Derive the agent's budget constraints for both periods (hint: the second period constraint is state-by-state).

(b) (XX Points) Reduce consumption in the utility function using the agent's budget constraints and derive the optimality conditions with respect to bond holdings, b_{t+1}, and asset holdings, a_{t+1}, taking as given prices and returns. (hint: the expectation operator E_t is linear (think of it as a probability weighted sum), so you can just go ahead and take derivatives inside the expectation operator).

(c) (XX Points) Use the optimality conditions to show that the price of the risky asset is a function of the price of the risk free bond, q_t, and the covariance of the stochastic discount factor, m_{t+1}, with the risky return on the asset,

$$p_t = \beta E_t[m_{t+1}]E_t[x_{t+1}] + \beta Cov_t(m_{t+1}, x_{t+1}), \quad m_{t+1} \equiv \left(\frac{c_{t+1}}{c_t}\right)^{-\theta}.$$

What factors drive the agent's valuation of the asset?

Exercise X:
Some More Long Questions ... (XX Points)