Final Exam
ECON 4310, Fall 2016

1. Do **not write with pencil**, please use a ball-pen instead.
2. Please answer in **English**. Solutions without traceable outlines, as well as those with unreadable outlines **do not earn points**.
3. Please start a **new page** for **every** short question and for every subquestion of the long questions.

Good Luck!

	Points	**Max**
Exercise A		**60**
Exercise B		**60**
Exercise C		**60**
Σ		**180**

Grade: __________

Exercise A:
Short Questions (60 Points)

Answer each of the following short questions on a seperate answer sheet by stating as a first answer True/False and then give a short but instructive explanation. You can write, calculate, or draw to explain your answer. You only get points if you have stated the correct True/False *and* provided a correct explanation to the question. We will not assign negative points for incorrect answers.

Exercise A.1: (10 Points) Labor supply in a static competitive equilibrium

Consider a static economy with a representative consumer that has the following preferences over consumption, c, and labor supply, h,

$$u(c,h) = \log(c) + \log(2-h),$$

and is subject to the budget constraint

$$c = wh,$$

where w is the wage rate per unit of labor supplied. The optimal labor supply is then independent of the wage rate and given by

$$h = 1.$$

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!
Your Answer:

True: ☒ False: ☐

The Lagrangian is given by

$$u(c,h) = \log(c) + \log(2-h) + \lambda(wh - c),$$

with the first order condition on labor supply:

$$\frac{1}{2-h} = \lambda w = \frac{1}{c} w.$$

This implies

$$c = w(2-h).$$

Combined with the budget constraint, we can have $2w = 2wh$, or $h = 1$.

Exercise A.2: (10 Points) Solow model with worker heterogeneity

Consider the following version of the Solow model with two types of workers, type 1 with productivity A_1 and type 2 with productivity A_2. The number of workers are given by L_1 and L_2, respectively. There are no population growth or technology growth. The production function is specified in (2), and the economy is described by the following equations

$$K_{t+1} = sY_t + (1-\delta)K_t \tag{1}$$

$$Y_t = \left[K_t^\rho + (A_1 L_1)^\rho + (A_2 L_2)^\rho\right]^{1/\rho}, \quad \rho > 0, \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of physical capital, and the saving rate s is constant. A_1, A_2 and L_1, L_2 are also constant. Denote the share of type 1 workers as $\theta \equiv \frac{L_1}{L} = \frac{L_1}{L_1+L_2}$.

The steady-state level of the capital stock per capita, $k_t \equiv K_t/L$, is described by

$$\delta k^\star = s\left[(k^\star)^\rho + (A_1\theta)^\rho + (A_2(1-\theta))^\rho\right]^{1/\rho}.$$

True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!

Your Answer:

True:☒ False: ☐

Directly use the definition of $k_t \equiv K_t/L$, and use the two equations

$$K_{t+1}/L = sY_t/L + (1-\delta)K_t/L \tag{3}$$

$$Y_t/L = \left[(K_t/L)^\rho + ((A_1L_1)/L)^\rho + ((A_2L_2)/L)^\rho\right]^{1/\rho}, \tag{4}$$

we can show the statement is correct.

Exercise A.3: (10 Points) The Intertemporal Elasticity of Substitution

Consider the optimal intertemporal consumption choice of a household in discrete and infinite time. The representative household has preferences $U = \sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta \in (0, 1)$ is the discount factor and the momentary utility function is

$$u(c_t) = \frac{c_t^{1-1/\theta} - 1}{1 - 1/\theta}, \quad 0 < \theta < 1.$$

The optimal behavior is characterized by the following consumption Euler equation

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{1}{1 + r_{t+1}}.$$

The Intertemporal Elasticity of Substitution (IES) is defined as

$$\text{IES} \equiv \frac{\partial \log(c_{t+1}/c_t)}{\partial \log(1 + r_{t+1})}.$$

Then the IES is equal to θ. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation!
Your Answer:

True: ☒ False: ☐

First, the marginal utility implied by the functional form of the utility function is

$$u'(c) = c^{-1/\theta},$$

and the consumption Euler equation then reads

$$\beta \left(\frac{c_{t+1}}{c_t} \right)^{-1/\theta} = (1 + r_{t+1})^{-1},$$

Take logs on both sides of the equation

$$\log \beta - \frac{1}{\theta} \log \left(\frac{c_{t+1}}{c_t} \right) = -\log(1 + r_{t+1}),$$

to yield

$$\text{IES} = \frac{\partial \log(c_{t+1}/c_t)}{\partial \log(1 + r_{t+1})} = \theta.$$

Exercise A.4: (10 Points) Ramsey model and Golden Rule capital stock

Consider the capital accumulation equation of the Ramsey model with constant population and without technology growth

$$k_{t+1} - k_t = k_t^\alpha - c_t - \delta k_t,$$

where k_t is the per capita capital stock, and c_t per capita consumption, $\alpha \in (0,1)$ the capital income share in the economy, $\delta \in (0,1)$ the depreciation rate of physical capital. The consumption Euler Equation is given by:

$$\frac{c_{t+1}}{c_t} = [\beta(1 + \alpha k_{t+1}^{\alpha-1} - \delta)]^{1/\theta}$$

where $0 < \beta < 1$ is the discount factor.
The Golden Rule per capita capital stock (the capital stock per capita that maximizes steady-state consumption per capita) depends on the discount factor β. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation! Your Answer:

True: ☐ False: ☒

Solve the capital accumulation equation for consumption and impose the steady-state condition, $k_t = k_{t+1} = k$. The Golden Rule capital stock is then characterized by

$$k_{GR} = \arg\max_{k \geq 0} k^\alpha - \delta k,$$

with the associated optimality condition

$$0 = \alpha k_{GR}^{\alpha-1} - \delta \quad \rightarrow \quad k_{GR} = (\alpha/\delta)^{1/1-\alpha}$$

Exercise A.5: (10 Points) Income, substitution and wealth effect

Consider a representative consumer who lives for only two periods denoted by $t = 1, 2$. The consumer is born in period 1 without any financial assets and leaves no bequests or debt at the end of period 2. The consumer's labor income is w_t in each period and her preferences over consumption can be represented by the utility function

$$U(c_1, c_2) = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1, \tag{5}$$

where the momentary utility function is given by

$$u(c) = \begin{cases} \frac{c^{1-\theta}-1}{1-\theta}, & \theta \neq 1. \\ \log(c), & \theta = 1, \end{cases}$$

with $\theta > 0$. The optimal consumption in period 1 is given by

$$c_1 = \frac{1}{1 + \beta^{1/\theta}(1+r)^{1/\theta - 1}} \left(w_1 + \frac{w_2}{1+r} \right).$$

If $\theta = 1$ and $w_2 = 0$ the consumption/saving decision is independent on the interest rate. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation! Your Answer:

True: ☒ False: ☐

The derivative of c_1 with respect to $1 + r$ reads

$$\begin{aligned} \partial c_1 / \partial (1+r) = & -\frac{(1/\theta - 1)\beta^{1/\theta}(1+r)^{1/\theta - 2}}{\left[1 + \beta^{1/\theta}(1+r)^{1/\theta - 1}\right]^2} \left(w_1 + \frac{w_2}{1+r} \right) \\ & - \frac{1}{1 + \beta^{1/\theta}(1+r)^{1/\theta - 1}} \frac{w_2}{(1+r)^2} \end{aligned}$$

Easy to see that it is independent on r if $\theta = 1$ (income and substitution effect cancel each other, first term above equal to 0) and $w_2 = 0$ (no wealth effect, second term above equal to zero).

Exercise A.6: (10 Points) Norwegian Fiscal Policy

Consider a small open economy populated by non-overlapping generations living one period and by an infinitely lived government, endowed with asset $a_0 = A$ at time $t = 0$. Each generation is subject to the budget constraint:

$$c_t = w_t + T_t$$

and the government is subject to the period-by-period budget constraint:

$$a_{t+1} = (1+r)a_t - T_t$$

where c_t is private consumption, w_t is exogenous private income, a_t is government's net saving, T_t is a public transfer from the government to the generation t, and r is the constant interest rate.
The government follows the following fiscal rule:

$$a_{t+1} = A \quad \forall t.$$

The consumption level of all generations will then be the same (i.e. $c_{t+1} = c_t \quad \forall t$), irrespective on wage growth. True or false?

Your Answer:

True: ☐ False: ☐

Don't forget the explanation! Your Answer:

True ☐ False: ☒

By plugging-in the fiscal rule into the Gov. Budget constraint:

$$T_t = rA \quad \forall t,$$

so that (using the private budget constraint) private consumption is

$$c_t = w_t + rA \quad \forall t.$$

Clearly consumption across generations is constant only if wage is constant over time.

Exercise B:
Long Question (60 Points)

A comparison of the Ramsey model and the OLG model

Consider a closed economy without exogenous technology and population growth, where firms produce a generic good Y_t with the production function

$$Y_t = F(K_t, L) = K_t^{\alpha} L^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K_t is aggregate capital and L is the number of workers in the economy. Capital depreciates at the rate $0 < \delta < 1$. For simplicity, let aggregate labor supply (population) be equal to one, $L = 1$, such that per capita variables, $x_t = X_t/L$, are the same as aggregate variables, $X_t = x_t = X_t L$.

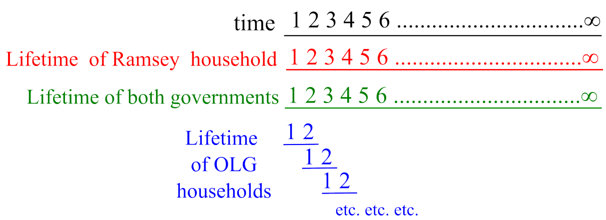
Consider now two different models. The Ramsey model where households are infinitely lived; and the OLG model where households live two periods, and where in each period two different generations are alive (young and old).
In both models agents chose consumption and asset holdings in order to maximize discounted lifetime utility:

$$\text{Ramsey:} \quad \sum_{t=1}^{T} \beta^t u(c_t)\,; \quad \text{OLG:} \quad u(c_1) + \beta u(c_2); \quad u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \quad \theta > 0$$

Denote with a_t the net asset holding of an household at the beginning of period t, that is equal to her net saving in $t-1$. Every period the household earns an interest $r_t a_t$ from her net assets holdings, and a wage w_t (the labor supply is exogenously set to 1 unit), and she is subject to a lump-sum tax τ_t. At birth each household is endowed with zero assets (in Ramsey $a_1 = 0$; in OLG $a_t = 0$ for an household born at t).
In both models markets are competitive, thus the prices of input factors are equal to their marginal products.
Both models are also populated by an infinitely-lived government who uses taxes τ_t and public debt D_t to finance government expenditure G_t. The government is subject to the period-by-period government budget constraint:

$$G_t = \tau_t + D_{t+1} - (1 + r_t) D_t$$

and the initial public debt is zero, i.e. $D_1 = 0$.
The timing is summarized in the following timeline:



(a) (5 Points) Write the period-by-period private budget constraint in both models.
Solution:

Allocation of points: 2.5 for Ramsey, 2.5 for OLG.

In Ramsey:

$$c_t + a_{t+1} = w_t + (1 + r_t)a_t - \tau_t \quad \forall t \in (1, \infty) \tag{6}$$

In OLG:

$$c_1 + a_2 = w_1 - \tau_1 \quad \text{when young} \tag{7}$$

$$c_2 + a_3 = w_2 + (1 + r_2)a_2 - \tau_2 \quad \text{when old} \tag{8}$$

where the timing refers to the timing within each generation.

(b) (15 Points) In the Ramsey model the intertemporal private budget constraint in net present value (NPV) terms is the following:

$$\sum_{t=1}^{\infty} \frac{c_t}{\Pi_{s=1}^{t}(1 + r_s)} = \sum_{t=1}^{\infty} \frac{w_t - \tau_t}{\Pi_{s=1}^{t}(1 + r_s)} - \lim_{T\to\infty} \frac{a_{T+1}}{\Pi_{s=1}^{T}(1 + r_s)}$$

Do the following:

- Impose the appropriate No-Ponzi condition in the equation above.
- Derive the intertemporal private budget constraint in net present value (NPV) terms in the OLG model, and impose the appropriate terminal condition.

Solution:

Allocation of points: 5 3 for NPV bc in OLG;5 for No-Ponzi, and 5 for terminal condition.

The no-Ponzi condition:

$$\lim_{T\to\infty} \frac{a_{T+1}}{\Pi_{s=1}^{T}(1 + r_s)} = 0. \tag{9}$$

In the OLG model, impose terminal condition $a_3 = 0$, and then:

$$c_1 + \frac{c_2}{1 + r_2} = w_1 - \tau_1 + \frac{w_2 - \tau_2}{1 + r_2} \tag{10}$$

where the timing refers to the timing within each generation.

(c) (5 Points) Write down the intertemporal government budget constraint in net present value (NPV) terms for both models, and assume that the time path of government debt is such that it is growing at a lower rate than the interest rate, i.e assume the following:

$$\lim_{T\to\infty} \frac{D_{T+1}}{\Pi_{s=1}^{T}(1 + r_s)} = 0.$$

Solution:

Allocation of points: to get full points debt must disappear from the final equation.

In both models (solve by iterative substitutions):

$$\sum_{t=1}^{\infty} \frac{\tau_t}{\Pi_{s=1}^{t}(1+r_s)} = \sum_{t=1}^{\infty} \frac{G_t}{\Pi_{s=1}^{t}(1+r_s)},$$

and the timing refers to the time of the world.

(d) (15 Points) Suppose that the sequence $\{\tau_t, G_t\}_{t=0}^{\infty}$ satisfies all the constraints and conditions imposed so far.
At t_1 the government decides a one-time unexpected increase in government expenditures ΔG_{t_1}. This increase can be financed with higher taxes $\Delta\tau_{t_1}$ OR higher debt ΔD_{t_1+1}.
Assume the households have perfect foresight, and anticipate the government budget constraint.
Is the household's optimal consumption decision different depending on whether ΔG_t is fully financed via the tax increase, or fully financed via a debt increase above?

- Answer the question above for the Ramsey model.
- Answer the question above for the OLG model.
- Motivate your answers.

Solution:

Allocation of points: 5 for Ramsey, and 5 for OLG; 5 for explanation. Full points require mention to Ricardian Equivalence.

In Ramsey, how G is financed does not matter for consumption.
In OLG, it does: consumption of old agents at t_1 changes only if the increase is financed via tax; whether consumption of young at t_1 change or not depends not only on τ_{t_1}, D_{t_1+1}, but also on τ_{t_1+1}, D_{t_1+2}. In general any increase in G_{t_1} financed via a sequence $\{\tau_t, D_{t+1}\}_{t=t_1}^{\infty}$ that implies financing via public debt which is due after the death of the agent alive at t_1 does not affect consumption (substitute the gbc NPV into the bc NPV).
Ricardian equivalence holds only in Ramsey but not in OLG: increase in debt is simply an increase in future taxation for this government. However, life-time horizon of OLG households is short, and they may not be around when it is time to pay the bill.

(e) (5 Points) Derive the equilibrium prices of labor (w_t) and the equilibrium price of capital (R_t) in both models.
(hint: in equilibrium aggregate demand and supply of labor are equal.) **Solution:**

Allocation of points: 2.5 for each price.
Marginal products of the respective factors:

$$R_t = \alpha(k_t/1)^{\alpha-1}$$
$$w_t = (1-\alpha)(k_t/1)^{\alpha},$$

where market clearing the labor market has been used to set $n = L = 1$

(f) (5 Points) Focus now on the Ramsey model. Market clearing in the capital market requires $a_t = k_t + D_t \quad \forall t$, and market clearing in the labor market requires that both demand and supply of labor are equal to 1. Suppose both the period-by-period government budget constraint and the period-by-period private budget constraint hold. Show that this imply that the capital accumulation equation in the Ramsey model holds:

$$k_{t+1} - k_t = k_t^{\alpha} - \delta k_t - c_t - G_t$$

(hint: recall $R_t = r_t + \delta$)
Solution:

Allocation of points: 2 points if they state capital market clearing condition right; 2 points if they use correctly the period-by-period budget constraints; 1 point if they use correctly the price of factors.

Use the clearing condition in capital market.

$$\begin{aligned} k_{t+1} = &\ a_{t+1} - D_{t+1} \\ = &\ w_t + (1+r_t)a_t - \tau_t - c_t - G_t + \tau_t - (1+r_t)D_t \\ = &\ (1+r_t)(a_t - D_t) + w_t - c_t - G_t \\ = &\ (1 + \alpha k_t^{\alpha-1} - \delta)k_t + (1-\alpha)k_t^{\alpha} - c_t - G_t \end{aligned}$$

where in the second line I used both period-by-period budget constraints, and in the fourth line I used the fact that wage is equal to the marginal product of labor, and the interest rate is the marginal product of capital minus the depreciation rate. Finally some simplification yields the equation above.

(g) (10 Points) Focus now on the OLG model. Market clearing in the capital market requires $a_t = k_t + D_t \quad \forall t$. Assume only the young can supply labor (i.e. $w_2 = 0$). Suppose both the period-by-period government budget constraint and the period-by-period private budget constraint hold. Show that this imply that the capital accumulation equation in the OLG model holds:

$$k_{t+1} - k_t = k_t^{\alpha} - \delta k_t - (c_t^O + c_t^Y) - G_t$$

where c_t^Y is consumption of the young at time t, and c_t^O consumption of the old at time t.

Solution:

Allocation of points: 4 points if they state capital market clearing condition right; 4 points if they use correctly the period-by-period budget constraints; 2 point if they use correctly the price of factors. Denote a_{t+1}^Y is the saving of the young generation at t, use the fact that $a_{t+1} = a_{t+1}^Y$, since the young generation at $t+1$ is born with 0 assets.

$$
\begin{aligned}
k_{t+1} = \quad & a_{t+1} - D_{t+1} \\
= & a_{t+1}^Y \quad - D_{t+1} \\
& = w_t^Y + (1+r_t)a_t^Y - \tau_t^Y - c_t^Y \\
& - [G_t - \tau_t + (1+r_t)D_t] \\
& = (1-\alpha)k_t^\alpha + (1+r_t)a_t^Y - \tau_t^Y - c_t^Y \\
& - [G_t - \tau_t + (1+r_t)D_t] \\
& = (1-\alpha)k_t^\alpha + (1+r_t)(k_t - D_t - a_t^O) - \tau_t^Y - c_t^Y \\
& - [G_t - \tau_t + (1+r_t)D_t]
\end{aligned}
$$

use $a_t^O(1+r_t) - \tau_t^O = c_t^O$, and $\tau_t^O + \tau_t^Y = \tau_t$

$$
\begin{aligned}
k_{t+1} & = (1-\alpha)k_t^\alpha + (1+r_t)(k_t - D_t - a_t^O) - \tau_t^Y - c_t^Y \\
& - [G_t - \tau_t + (1+r_t)D_t] \\
& = (1-\alpha)k_t^\alpha + (1+r_t)k_t - (c_t^O + \tau_t^O) - D_t(1+r_t) - \tau_t^Y - c_t^Y \\
& - [G_t - \tau_t + (1+r_t)D_t] \\
& = (1-\alpha)k_t^\alpha + (1+r_t)k_t - (c_t^O + c_t^Y) - G_t \\
& = (1-\alpha)k_t^\alpha + (1+\alpha k_t^{\alpha-1} - \delta)k_t - (c_t^O + c_t^Y) - G_t \\
& \Rightarrow k_{t+1} - k_t = k_t^\alpha - \delta k_t - (c_t^O + c_t^Y) - G_t
\end{aligned}
$$

Exercise C:
Long Question (60 Points)

A real business cycle model

Consider a representative household of a closed economy. The household has a planning horizon of two periods and is endowed with the following preferences over consumption, c,

$$U = u(c_1) + \beta \mathrm{E} u(c_2(s_2)),$$

with the following marginal utility

$$u'(c) = c^{-1}, \text{ (log-utility)}.$$

The variable s_2 denotes the state of the economy in the second period which follows the stochastic process

$$s_2 = \begin{cases} s_G, \text{ with prob. } p \\ s_B, \text{ with prob. } 1-p, \end{cases}$$

and the household conditions the consumption, $c_2(s_2)$, in the second period on the state, s_2. Assume the household's labor supply is exogenous and always equal to 1.

Labor market assumptions:

Assume that in each period and in each state of the economy, s_t, there is a linear (in labor n_t) production technology of the form

$$y_t(s_t) = A_t(s_t) n_t(s_t),$$

and the labor market is assumed to be competitive. Assume the labor productivity in the first period is given by $A_1 = A$, and the labor productivity is higher in the good state of the second period,

$$A_2(s_G) = A + A(1-p)\sigma > A_2(s_B) = A - Ap\sigma, \quad \sigma > 0, A > 0, 0 < p < 1,$$

than in the bad state of the second period. Notice that the expected productivity in the second period is the same as the productivity in the first period, $E[A_2(s_2)] = A_1 = A$. The wages are denoted as w_1, $w_2(s_G)$, and $w_2(s_B)$.

Asset market assumptions:

Assume the household does have access to a risk-free asset, a_2, and the associated interest rate is denoted as r_2.

(a) (5 Points) Find the equilibrium wages, w_1, $w_2(s_G)$, and $w_2(s_B)$.

Solution:

Since labor markets are competitive, we should have wages equal to productivity as follows:

$$w_1 = A,$$
$$w_2(s_G) = A_2(s_G),$$
$$w_2(s_B) = A_2(s_B).$$

***XX Allocation of points**:*
1 point for w_1; 2 points for each of the other quantities

(b) (5 Points) Write down the state-by-state budget constraints for the household.

Solution:

the state-by-state budget constraints for the household are:

$$c_1 + a_2 = w_1,$$
$$c_2(s_2) = w_2(s_2) + (1 + r_2)a_2, \ \forall s_2 \in S \equiv \{s_G, s_B\}.$$

***XX Allocation of points**:*
1 point for c_1; 2 points for each of the other quantities

(c) (10 Points) Let $(\lambda_1, \lambda_2(s_G), \lambda_2(s_B))$ denote the Lagrange multipliers of the state-by-state budget constraints. State the representative agent's Lagrangian. (Note that the expected utility for the second period is the summation of utility across good and bad states, weighted by probability, i.e., $\mathrm{E}u(c_2(s_2)) = pu(c_2(s_G)) + (1 - p)u(c_2(s_B))$.)

Solution:

The state-by-state budget constraints are:

$$c_1 + a_2 = w_1 \times 1,$$
$$c_2(s_2) = w(s_2) \times 1 + (1 + r_2)a_2, \ \forall s_2 \in S \equiv \{s_G, s_B\}.$$

The Lagrangian can be written in the state-ordered form as

$$\begin{aligned}\mathcal{L} = u(c_1) &+ \lambda_1 [w_1 - a_2 - c_1] \\ &+ \beta p [u(c_2(s_G))] + \lambda_2(s_G) [w(s_G) + (1 + r_2)a_2 - c_2(s_G))] \\ &+ \beta(1 - p) [u(c_2(s_B))] + \lambda_2(s_B) [w(s_B) + (1 + r_2)a_2 - c_2(s_B))] .\end{aligned}$$

XX Allocation of points:
10 points for the Lagrangian: if correct or make sense (could be in other formulations); deduct 3 points if one of the three blocks wrong.

(d) (10 Points) Derive the optimality conditions with respect to consumption, $(c_1, c_2(s_G), c_2(s_B))$ and savings, a_2 by using multipliers.

Solution:

The optimality conditions with respect to the choices are:

$$0 = \frac{\partial \mathcal{L}}{\partial c_1} = u'(c_1) - \lambda_1 \tag{11}$$

$$0 = \frac{\partial \mathcal{L}}{\partial c_2(s_G)} = \beta p u'(c_2(s_G)) - \lambda_2(s_G) \tag{12}$$

$$0 = \frac{\partial \mathcal{L}}{\partial c_2(s_B)} = \beta(1-p) u'(c_2(s_B)) - \lambda_2(s_B) \tag{13}$$

$$0 = \frac{\partial \mathcal{L}}{\partial a_2} = -\lambda_1 + [\lambda_2(s_G) + \lambda_2(s_B)](1 + r_2). \tag{14}$$

XX Allocation of points:
Deduct 2.5 point if the FOC for one of the choices wrong.

(e) (10 Points) Derive the stochastic consumption Euler equation (only involves with $c_1, c_2(s_2), \beta$ and r_2 and No multipliers).

Solution:

The stochastic consumption Euler equation is given by:

$$u'(c_1) = \beta \mathrm{E}\left[u'(c_2(s_2))\right](1 + r_2). \tag{15}$$

XX Allocation of points:
10 points if the final formula correct; otherwise, 0 points.

(f) (10 Points) For (f) and (g), assume that the asset a_2 is available in zero supply.

What is the household's optimal choice of a_2 in the equilibrium? What are the household's optimal choices of consumption? Can the household fully smooth consumption? i.e., are c_1, $c_2(s_G)$ and $c_2(s_B)$ equal?

Solution:

In equilibrium $a_2 = 0$ since we assume this is a representative household and zero net asset supply. The state-by-state budget constraints imply the following consumption levels

$$\begin{aligned} c_1 &= w_1 \\ c_2(s_2) &= w(s_2),\ \forall s_2 \in S, \end{aligned}$$

or,

$$\begin{aligned} c_1 = w_1 &= A, \\ c_2(s_G) = w_2(s_G) &= A_2(s_G) > A, \\ c_2(s_B) = w_2(s_B) &= A_2(s_B) < A. \end{aligned}$$

so consumptions are not fully smoothed.

***XX Allocation of points**:*
4 points to get the savings correct. 2 points for each of the consumption quantities.

(g) (10 Points) Is the equilibrium interest rate r_2 higher or lower than $r_{RN} \equiv \frac{1}{\beta} - 1$? Why? (Hint: do it step by step: (1) use the budget constraint to link consumption and wages; (2) use the Euler equation and the result, $u'(w_1) \leq \mathrm{E}[u'(w(s_2))]$, which comes from the Jensen's inequality.)

Solution:

In the stochastic economy with $\sigma > 0$, Jensen's inequality implies

$$(1 + r_2)\beta = \frac{u'(c_1)}{\mathrm{E}\left[u'(c(s_2))\right]} = \frac{u'(w_1)}{\mathrm{E}\left[u'(w(s_2))\right]} = \frac{u'(\mathrm{E}\left[w(s_2)\right])}{\mathrm{E}\left[u'(w(s_2))\right]} < 1,$$

such that the interest rate in this economy is smaller than the risk neutral interest rate, $r_{RN} \equiv \frac{1}{\beta} - 1$.

***XX Allocation of points**:*

(1) 5 points: get the budget constraint and use the Euler equation correctly

(2) 5 points: use the inequality and compare the two interest rates correctly