Problem Set on Labor Supply and Competitive Equilibrium in Static Models [1]

Exercise 1: Labor Supply in Static Models (Those parts not covered in lectures)

a) Consider a worker who gains utility from consumption and leisure, $U(c, l)$. Suppose his wage rate is 100 NOK per hour. There are 168 hours in a week. Suppose that wages go up and this worker reduces labor supply.

(1) Illustrate (using a graph) the income effect and the substitution effect.

(2) In this case, is the elasticity of labor supply positive or negative? Explain.

b) Now suppose that after the wage increase, labor supply increases.

(1) Illustrate (using a graph) the income effect and the substitution effect.

(2) In this case, is the elasticity of labor supply positive or negative? Explain.

c) Yicheng's hourly wage is 50 NOK per hour and he can work up to 168 hours in a week. Suppose his utility function is U(c,l)=log(c) + log(l).

(1) How much will he work and how much will he consume?

(2) Suppose NAV introduces a welfare program that guarantees everyone 1500 NOK per week, but the takeback rate is 100%, i.e. if you make 100 NOK you only get 1400 from NAV. Draw Yicheng's new budget constraint. How many hours does Yicheng choose to supply now? Over what range of hours should we never expect to see Yicheng working? How many hours must Yicheng work to be off of the welfare program?

(3) Now the government reduces the takeback rate from 100 percent to 20 percent. Draw Yicheng's new budget constraint and determine how many hours Yicheng chooses to work now. Are there any ranges in which we should not expect to see Yicheng working at all? In reducing the takeback rate, how does the government affect the work incentives of different types of people? For example, people who chose to work zero hours per week when the takeback rate was 100 percent and people who chose to stay off of the program when the takeback rate was 100 percent.

d) Marcus' indifference curves in leisure-consumption space (leisure on the x axis) are steeper than Yicheng's. True or False: Marcus must be offered higher wages than Yicheng to work the same amount of hours?

[1] We are grateful to Andreas Muller for the materials in the 2nd part of this problem set. Contact: Hans Holter: hans.holter@econ.uio.no; Yicheng Wang: wangyicheng1192@gmail.com.

Exercise 2: A Static Competitive Equilibrium

Consider a static economy with a representative consumer that has the following preferences over consumption, c, leisure, l, and a public good, g,

$$u(c, l, g) = \log\left(c - \psi(1-l)^{\theta}\right) + \log(g), \quad \theta > 1,$$

and is subject to the budget constraint

$$c = (1 - \tau^n)(1 - l)w + rk_0.$$

where τ^n is a proportional tax rate on labor income, and k_0 denotes the consumers' initial endowment of physical capital. The representative firm produces consumable output, y, with the following technology

$$y = k^{\alpha} n^{1-\alpha}, \quad 0 < \alpha < 1,$$

by renting physical capital, k, from consumers at the rental rate r, and labor, n, at the wage rate w. The government spends a fixed fraction $\gamma = g/y$ of output on public goods by setting labor income taxes to balance the government's budget

$$\tau^n n w = g.$$

Remember the definition of a competitive equilibrium with a public sector:

> A **competitive equilibrium** is an allocation $\{c, l, k, n\}$ and a set of prices and taxes $\{r, w, \tau^n\}$ such that
>
> (1) The representative consumer chooses c and l to maximize utility subject to the private budget constraint, taking as given prices and taxes.
>
> (2) The representative firm chooses physical capital k and labor input n to maximize profits, taking as given prices.
>
> (3) The government chooses tax policy τ^n to balance the government budget.
>
> (4) The markets for goods, capital, and labor clear.

In what follows, we will compute the competitive equilibrium of this economy step by step.

(a) Solve the consumers maximization problem.

(b) Derive the optimality conditions of the firm's maximization problem and show that it will make zero profit.

(c) Find the equilibrium prices $\{r, w\}$ that clear the labor and the capital market for a given tax rate τ^n.

(d) Show that the wage income of the consumer is equal to a constant share of output, $wn = (1 - \alpha)y$. What is the equilibrium tax rate τ^n of this economy that balances the government budget?

(e) Verify that Walras' law holds in the competitive equilibrium derived above, i.e., check whether the goods market clears at equilibrium prices and taxes

$$y(\tau^n) = c(\tau^n) + g.$$