Problem Set 2:¹ Introduction

Exercise 1: Immigration in the Solow model

Consider a closed economy with a neoclassical production function, exogenous technological progress, A_t , a fixed saving rate, s, and a constant labor force, L, as described by the following equations (the Solow model)

$$K_{t+1} - K_t = sY_t - \delta K_t \tag{1}$$

$$Y_t = K_t^{\alpha} (A_t L)^{1-\alpha}, \, 0 < \alpha < 1,$$
(2)

$$A_{t+1} = (1+g)A_t, \, A_0 > 0,$$

where $0 \le \delta \le 1$ is the depreciation rate of physical capital (in the lecture we have abstracted from depreciation for simplicity).

- (a) Remove the trend from Equations (1) and (2) by writing all endogenous variables X_t in terms of efficiency units $x_t \equiv X_t/(A_tL)$.
- (b) Compute the wage rate, w_t , and the rental rate of capital, r_t , (the interest rate of this economy will be $r_t \delta$) in this economy.
- (c) Compute the stable steady-state capital stock per efficiency unit, $k^* > 0$, of this economy.
- (d) Show that the capital stock per efficiency unit, k_t , is increasing over time as long as $0 < k_t < k^*$ (hint: look at the ratio k_{t+1}/k_t in the capital accumulation equation to answer this question).
- (e) Suppose the economy is in a steady-state. What happens to the aggregate output, the wage rate, and the rental rate on impact if the number of workers in the economy increases by ∂L due to immigration?
- (f) What happens to aggregate output, the wage rate, and the rental rate over time after the immigration wave?
- (g) Would the wage rate be higher in the long-run if there had been no immigration?

¹We are grateful to Andreas Muller for the materials in this problem set. Contact: Hans Holter:hans.holter@econ.uio.no; Yicheng Wang: wangyicheng1192@gmail.com.