Problem Set 5: More on Ramsey's Growth Model

Public spending in Ramsey's growth model

In this exercise we will study at a discrete-time version of Ramsey's growth model again. The economy is closed and we consider a representative agent with the following preferences over consumption

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where c_t denotes period t consumption and $\beta \in (0,1)$ is the subjective discount factor. The momentary utility function is of the form

$$u(c_t) = \frac{c_t^{1-\theta} - 1}{1-\theta},$$

with $\theta > 1$. Every period the agent earns a wage w_t (the labor supply is exogenously set to 1 unit), an interest $r_t a_t$ from her assets holdings and she is subject to the lump-sum tax τ_t. In equilibrium, the agent will choose the sequence consumption and asset holdings $\{c_t, a_{t+1}\}_{t=0}^{\infty}$ to maximize U subject to the period-by-period budget constraint

$$c_t + a_{t+1} = w_t + (1 + r_t)a_t - \tau_t, \tag{2}$$

for a given a_0. The agent is atomic and her decisions do not influence aggregate variables, thus she takes the sequence of taxes, wage rates and interest rates as given.

The representative firm demands physical capital k_t and labor n_t to produce output y_t with the Cobb-Douglas technology

$$y_t = k_t^{\alpha} n_t^{1-\alpha}. \tag{3}$$

The firm is atomic and acts as a price-taking profit maximizer. Capital can be rented at the rental rate $R_t = r_t + \delta$ (note that the depreciation rate δ is the difference between the rental rate and the interest rate) while labor costs w_t.

The government can raise lump-sum taxes τ_t and rolls over debt in the form of one-period bonds, D_{t+1}, to finance government expenditure, G_t. As it pays an interest rate r_t on the outstanding debt, D_t, the government faces a period-by-period budget constraint

$$G_t = \tau_t + D_{t+1} - (1 + r_t)D_t. \tag{4}$$

Moreover, assume that the time path of government debt is such that it is growing at a lower rate than the interest rate

$$\lim_{T \to \infty} \frac{D_{T+1}}{\Pi_{s=0}^{T}(1 + r_s)} = 0.$$

In other words, it is not feasible for the government to finance the outstanding debt (plus interest payments) by issuing ever more debt as time goes by.

The first welfare theorem applies to this economy such that the competitive equilibrium is efficient in the Pareto sense. Thus, we know that the solution to the social planner's problem (which characterizes the Pareto efficient allocation) is equivalent to the competitive market equilibrium. According to the social planner's solution, the same consumption Euler equation and resource constraint (goods market clearing) along with the so-called transversality condition (which stands in for the no-Ponzi condition)

$$\frac{c_{t+1}}{c_t} = [\beta(1+r_{t+1})]^{1/\theta} = \left[\beta(1+\alpha k_{t+1}^{\alpha-1}-\delta)\right]^{1/\theta}$$

$$k_{t+1} - k_t = k_t^{\alpha} - \delta k_t - c_t - G_t$$

$$\lim_{t\to\infty} \beta^t c_t^{-\theta} k_{t+1} = 0$$

determine the optimal solution of the dynamic system. Let us assume that $G_t = G$, then we can define two correspondances. One which characterizes all possible combinations of (c_t, k_t) when consumption is constant,

$$\mathcal{C}_1(k) \equiv \left\{c \in [0,\infty) : c_{t+1}/c_t = \left[\beta(1+\alpha k^{\alpha-1}-\delta)\right]^{1/\theta}, \, c_{t+1} = c_t = c\right\},$$

and one which captures all combinations if the physical capital stock is constant,

$$\mathcal{C}_2(k) \equiv \{c \in [0,\infty) : c = k_t^{a} - (k_{t+1} - (1-\delta)k_t) - G, \, k_{t+1} = k_t = k\}.$$

Assume that the economy is in a stationary equilibrium with constant government expenditures, $G_t = G$, and tax policy, $\tau_t = \tau$.

(a) Consider an unexpected and temporary cut of ΔG in government expenditures from period t_0 until period $t_1 = t_0 + 1$. Sketch the dynamics of consumption and physical capital back to the steady-state in the phase diagram. Sketch the time path of the wage rate and interest rates.

(b) Consider an unexpected and temporary cut of ΔG in government expenditures from period t_0 until period $t_1 = t_0 + T, T > 1$. Sketch the dynamics of consumption and physical capital back to the steady-state in the phase diagram. Sketch the time path of the wage rate and interest rates.

(c) Consider an unexpected and permanent cut of ΔG in government expenditures for all future periods $t \geq t_0$. Sketch the dynamics of consumption and physical capital to the new steady-state in the phase diagram. Sketch the time path of the wage rate and interest rates.

(d) Consider instead an unexpected and temporary decrease of $\Delta\beta$ in the discount factor from period t_0 to period t_1 (think of t_1 could be 1, or T, or $+\infty$). Sketch the dynamics of consumption and physical capital back to the steady-state in the phase diagram.