Problem Set 8:
Optimal Fiscal Policy (Solution)

Exercise: The Norwegian Handlingsregelen

Consider a small open economy populated with non-overlapping generations of households that live for one period. The size of each generation is one, and the generation living in period t earns an exogenously given wage w_t. The government of the economy is endowed with initial resources (due to an oil windfall, for example) of value

$$B = -b_0$$

where b_0 denotes the initial debt position of the government as in previous problem sets (negative debt can be interpreted as assets). The government can impose transfers T_t on each generation to redistribute resources across generations, such that the period-by-period budget constraint of the generation living in period t reads

$$c_t = w_t + T_t, \tag{1}$$

where c_t denotes the consumption level of each generation. The period-by-period budget constraint of the infinitely-lived government reads

$$b_{t+1} = (1+r)b_t + T_t, \tag{2}$$

where r denotes the exogenous interest rate on the international capital market (which is assumed to be constant for the ease of exposition). Without imposing any further restrictions on fiscal policy (except a no-Ponzi condition of course), the net present value budget constraint of the government reads

$$\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^{t+1}} = B, \tag{3}$$

such that the present value of all transfers cannot exceed the value of initial assets, B. The government is benevolent towards present and future generations and maximizes a welfare function equal to a weighted sum of each generation's utility

$$\sum_{t=0}^{\infty} \beta_t u(c_t), \quad \beta_0 = 1, \tag{4}$$

where β_t (not to be confused with the discount factor β^t, where t denotes the power of β) denotes the welfare weight that the government puts on each generation t.

(a) State the optimality conditions of the government's decision problem (hint: reduce consumption from the problem before maximizing the objective)

$$W_t = \max_{\{c_t, T_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_t u(c_t) \text{ s.t. } (1), (3).$$

Why does the Ricardian equivalence proposition not apply to this economy?
Solution:

Reduce consumption in the welfare function by plugging in Equation (1) and derive the optimality condition with respect to transfers, T_t,

$$0 = \beta_t u'(w_t + T_t) - \frac{\lambda}{(1+r)^{t+1}}, \quad \forall t,$$

where λ is the Lagrange multiplier on the present value budget constraint. There is an optimal level of transfers/taxes in this economy which implies that the Ricardian equivalence proposition does not hold. The reason is that agents have finite lives and are not altruistic towards other generations, such that all agents prefer running down assets and enjoy transfers as they are not around in the next period.

(b) Assume that marginal utility is given by $u'(c) = c^{-\theta}$, $\theta > 0$. Derive the government's Euler equation, by combining the optimality conditions of two subsequent generations, t and $t+1$, respectively.
Solution:

Combine the optimality condition for transfers to generation t with the one for generation $t+1$ to yield the government's Euler equation

$$\frac{\beta_t u'(c_t)}{\beta_{t+1} u'(c_{t+1})} = \frac{\lambda}{(1+r)^{t+1}} \frac{(1+r)^{t+2}}{\lambda} = 1 + r.$$

Using the functional form of marginal utility, $u'(c_t) = c_t^{-\theta}$, the Euler equation can be rewritten as

$$c_{t+1} = \left[\frac{\beta_{t+1}}{\beta_t} (1+r) \right]^{1/\theta} c_t. \tag{5}$$

(c) Solve for c_t as a function of c_0 using the government's Euler equation. Then, only for this subquestion, set the parameter $\theta = 1$ and derive the optimal level of consumption c_0 from Equations (1) and (3).
Solution:

Make use repeatedly of Equation (5) to yield

$$\begin{aligned}
c_t &= \left[\beta_t / \beta_{t-1} (1+r)\right]^{1/\theta} c_{t-1} \\
&= \left[\beta_t / \beta_{t-1} (1+r)\right]^{1/\theta} \left[\beta_{t-1} / \beta_{t-2} (1+r)\right]^{1/\theta} c_{t-2} \\
&= \left[\beta_t / \beta_{t-2} (1+r)(1+r)\right]^{1/\theta} c_{t-2} \\
&\vdots \\
&= \left[\beta_t / \beta_{t-t} (1+r)^t\right]^{1/\theta} c_{t-t} \\
&= \left[\beta_t / \beta_0 (1+r)^t\right]^{1/\theta} c_0
\end{aligned}$$

Then rewrite the net present value budget constraint

$$\sum_{t=0}^{\infty} \frac{T_t}{(1+r)^{t+1}} = \sum_{t=0}^{\infty} \frac{c_t - w_t}{(1+r)^{t+1}} = B,$$

and substitute out consumption to yield (henceforth we set $\theta = 1$)

$$\begin{aligned}\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^{t+1}} &= \sum_{t=0}^{\infty} \frac{\beta_t/\beta_0(1+r)^t c_0}{(1+r)^{t+1}} \\ &= \frac{c_0}{1+r}\left(\sum_{t=0}^{\infty} \beta_t/\beta_0\right) = B + \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^{t+1}}.\end{aligned}$$

The initial consumption level can then be written as

$$c_0 = (1+r)\left(\sum_{t=0}^{\infty} \beta_t/\beta_0\right)^{-1}\left(B + \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^{t+1}}\right),$$

and the consumption level for any generation t is given by

$$\begin{aligned}c_t &= \beta_t/\beta_0(1+r)^t c_0 \\ &= (1+r)^{t+1}\frac{\beta_t/\beta_0}{\sum_{t=0}^{\infty} \beta_t/\beta_0}\left(B + \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^{t+1}}\right).\end{aligned}$$

Note that the optimal consumption level only depends on the relative welfare weights (we chose β_0 as the basis, but you could choose any other β_t). By normalizing the basis $\beta_0 = 1$ we can simplify the expressions without loosing generality in the results.

(d) Consider the Norwegian Handlingsregelen which roughly state that fiscal policy is restricted to be

$$-b_{t+1} = B,$$

for all generations t. Or in words, the government is only allowed to take out the returns on the stock of assets, B. What transfer and private consumption pattern does this imply for each generation? What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

Solution:

First, plug the policy rule, $-b_{t+1} = B$, into the government's period-by-period budget constraint and solve for the implied sequence of transfers

$$b_{t+1} = (1+r)b_t + T_t \quad \Leftrightarrow \quad -B = -(1+r)B + T_t \quad \Leftrightarrow \quad T_t = rB.$$

This implies that the consumption of every generation will be given by

$$c_t = w_t + rB.$$

Use this in the Euler equation stated in Equation (5) to yield

$$\begin{aligned}
\beta_t &= \frac{1}{1+r}\left(\frac{c_t}{c_{t-1}}\right)^{\theta}\beta_{t-1} \\
&= \frac{1}{(1+r)(1+r)}\left(\frac{c_t}{c_{t-2}}\right)^{\theta}\beta_{t-2} \\
&\vdots \\
&= \frac{1}{(1+r)^t}\left(\frac{c_t}{c_{t-t}}\right)^{\theta}\beta_{t-t} \\
&= \frac{1}{(1+r)^t}\left(\frac{w_t+rB}{w_0+rB}\right)^{\theta}\beta_0, \quad \beta_0 = 1.
\end{aligned}$$

This equation characterizes the whole sequence of welfare weights β_t that would be consistent with the Norwegian Handlingsregelen.

(e) Let the wage growth be given by $w_{t+1}/w_t = (1+g)$. Suppose that the government followed instead the fiscal rule

$$-b_t/w_t = B/w_0,$$

for each generation t. Or in words, the government wants to keep the stock of assets as a fraction of wages constant. What sequence of welfare weights $\{\beta_{t+1}\}_{t=0}^{\infty}$ would correspond to this fiscal policy rule?

Solution:

First, plug this policy rule into the governments period-by-period budget constraint and solve for the implied sequence of transfers

$$-\frac{b_{t+1}}{w_{t+1}}\frac{w_{t+1}}{w_t} = -(1+r)\frac{b_t}{w_t} - \frac{T_t}{w_t} \quad \Leftrightarrow \quad \frac{B}{w_0}\frac{w_{t+1}}{w_t} = (1+r)\frac{B}{w_0} - \frac{T_t}{w_t},$$

such that

$$T_t = B\frac{w_t}{w_0}\left(1+r-\frac{w_{t+1}}{w_t}\right), \quad \frac{w_{t+1}}{w_t} = (1+g).$$

This implies that the consumption of every generation will be given by

$$c_t = w_t + B\frac{w_t}{w_0}(r-g) = w_t\left[1+\frac{B}{w_0}(r-g)\right].$$

The Euler equation implies that the corresponding welfare weights are given by

$$\begin{aligned}
\beta_t &= \frac{1}{(1+r)^t}\left(\frac{c_t}{c_0}\right)^{\theta}\beta_0 = \frac{1}{(1+r)^t}\left(\frac{w_t}{w_0}\frac{1+B/w_0(r-g)}{1+B/w_0(r-g)}\right)^{\theta}\beta_0 \\
&= \frac{1}{(1+r)^t}\left(\frac{(1+g)^t w_0}{w_0}\right)^{\theta}\beta_0 \\
&= \frac{\left[(1+g)^t\right]^{\theta}}{(1+r)^t}\beta_0, \quad \beta_0 = 1.
\end{aligned}$$

(f) Calculate the relative welfare weight β_{t+1}/β_t under both fiscal policy rules considered in parts (d) and (e). What policy rule puts a higher relative welfare weight on future generations?

Solution:

We can see from the Euler equation

$$\beta_{t+1} = \frac{1}{1+r}\left(\frac{c_{t+1}}{c_t}\right)^{\theta}\beta_t,$$

that the first policy rule where $c_t = w_t + rB$ yields

$$\frac{1}{1+r} < \beta_{t+1}^{\text{NOR}}/\beta_t^{\text{NOR}} = \frac{1}{1+r}\left(\frac{(1+g)w_t + rB}{w_t + rB}\right)^{\theta} < \frac{(1+g)^{\theta}}{1+r},$$

where the relative welfare weight $\beta_{t+1}^{\text{NOR}}/\beta_t^{\text{NOR}}$ is strictly increasing in w_t. Note that in the limit

$$\lim_{w_t\to\infty} \beta_{t+1}^{\text{NOR}}/\beta_t^{\text{NOR}} = (1+g)^{\theta}/(1+r),$$

as the size of the fund in comparison to the growing wages becomes negligible over time. The second policy rule where $c_t = w_t\left[1 + B/w_0(r-g)\right]$ yields

$$\beta_{t+1}/\beta_t = \frac{1}{1+r}\left(\frac{w_{t+1}}{w_t}\frac{1+B/w_0(r-g)}{1+B/w_0(r-g)}\right)^{\theta} = \frac{(1+g)^{\theta}}{1+r}.$$

Thus, the first policy rule is less favorable to future generations, as the government implicitly puts a lower relative welfare weight, $\beta_{t+1}^{\text{NOR}}/\beta_t^{\text{NOR}} < (1+g)^{\theta}/(1+r)$, on future generations. Also, note that a policy rule leading to

$$c_t = c_{t+1} = c$$

would imply a relative welfare weight of $1/(1+r)$ (this is the same as Nordhaus' discount factor, while $(1+g)^{\theta}/(1+r)$ would correspond to Stern's discount factor.) The Handlingsregelen are located somewhere in between and converge to Stern's discount factor over time. In a nutshell, a lower (higher) relative welfare weight than the Handlingsregelen means that you should extract more (less) than r percent from the value of the petroleum fund.
