# *UNIVERSITY OF OSLO*
# *DEPARTMENT OF ECONOMICS*

Exam: **ECON4310 – Intertemporal macroeconomics**

Date of exam: Wednesday, December 06, 2006 **Grades will be given: January 4, 2007**

Time for exam: 02:30 p.m. – 05:30 p.m.

The problem set covers 5 pages incl. cover sheet

The exam consists of three problems. They count as indicated. Start by reading through the whole exam, and make sure that you allocate time to answering questions you find easy.

Resources allowed:

- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

# 4310 – Intertemporal macroeconomics

## Exam, December 6, 2006

Before you start, please read the following:

- You can answer in either English or Norwegian.
- Answer all questions and write brief and concise answers!
- Allocate time spent on each question wisely.
- Good style will not matter for grades, but please write clearly.
- Good luck!

## 1 : True or false? (20%)

For each of the statements, true or false, explain why. *Be brief and concise!*

1. In his book from 1987, Robert Lucas calculated the cost of U.S. business cycles to be about 8% of consumption.
2. According to Kydland and Prescott (1990) "Real facts and a monetary myth", U.S. government expenditures have been countercyclical.
3. Storesletten, Telmer and Yaron (2001)'s findings support Lucas (1987)'s assumption that deviations from trend consumption are identically and independely distributed over time.
4. According to Barro (1974), even if agents are finitely-live, Ricardian Equivalence *may* still hold if every agent have bequest motive and leaves positive bequest to their childern.

## 2 : Overlapping Generations Economy (40%)

Consider a two-period overlapping generations model without production. At each time period $t = 0, 1, ...$, a new generation of size $L_t$ are born. Population grows at a constant rate $n \geq 0$, such that $L_t = (1+n)L_{t-1}$. Agents receive $w_t$ units of endowment in the period when they are born and no further endowment when they become old. Agents have access to a storage technology with constant net rate of return $r$. The preference and endowment structures of successive generations are identical. The preference of the representative member of generation $t$ are given by

$$U_t = \frac{c_{1t}^{1-\sigma} - 1}{1-\sigma} + \beta \frac{c_{2t+1}^{1-\sigma} - 1}{1-\sigma}$$

where $\sigma > 0$. Finally, assume that in the initial period, 0, in addition to the $L_0$ young individuals, there are $[1/(1+n)]\, L_0$ initial old individuals, which size is normalized to 1. The representative initial old has endowment $w_0$, with preference simply $c_{20}$.

1. For simplicity, assume $n = 0$ and $L_t = 1$, for all $t$. That is, population is constant and normalized to one.

   (a) Derive the individual's consumption function in both periods, and show how first-period consumption is related to the interest rate.

   (b) Assume initially the economy is stationary such that the endowment process is $w_t = w^*$ for all $t$. Suppose at some period $m$ there is a temporary reduction in $w_m$, from to $w^* - \varepsilon$. In the following period, $w_{m+1}$ is back to $w^*$. Show how the *aggregate* savings are affected in period $m$ and subsequently.

2. Now let $n > 0$ and $\sigma = 1$, i.e. logarthmic utility. Assume that there is a pay-as-you-go (PAYG) social security system in the economy. The young agents pay lump-sum taxes in the amount $\tau$ each period, and when they get old, they receive benefits in the amount $b$. Assume that the system is self-financing each period so that $b = (1+n)\,\tau$

   (a) Derive the individual's consumption function in both periods.

   (b) Describe the condition (between $n$ and $r$) under which introducing the PAYG social security system is Pareto improving at the balanced-growth path. (Hint: obtain the indirect lifetime utility function under social security, $V(\tau)$, and calculate $V'(0)$).

## 3 : Pricing a nominal bond(40%)

Consider a stochastic endowment economy which is populated by a large number of identical individuals. All individuals receive the same random endowment stream $\{y_t\}_{t=0}^{\infty}$. There also exists a market for one-period nominal bonds, i.e. bonds which promises the holder one unit of currency the next period.

Each individual's optimization problem is

$$\max_{\{c_t, b_t\}} \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$

subject to her/his period-by-period budget constraint.

$$c_t + \frac{q_t b_t}{p_t} = y_t + \frac{b_{t-1}}{p_t},$$

where $q_t$ is the nominal price of the bond, $b_t$ is the number of units of the nominal bond, and $p_t$ is the exogenously given price level in economy. $c_t$ and $y_t$ are both real variables.

1. Write down a representative individual's Bellman equation. What are the individual's control variable(s) and endogenous and exogenous state variables(s) at time $t-1$, $t$, and $t+1$.

   Find the representative individual's Euler equation by taking the first order condition(s) and envelope condition(s) of the Bellman equation.

   You might define the rate of inflation as $\pi_{t+1} \equiv p_{t+1}/p_t$.

2. Assume that inflation has constant conditional expected mean and variance, and that money is neutral both in the long and short run, i.e. monetary policy only affects nominal variables, like the price level, and has no impact on real variables like output, consumption or investment.

   The central bank has the option of two different monetary policy regimes:

   - Monetary Policy Regime 1: Inflation is *positively correlated* with aggregate consumption growth.
   - Monetary Policy Regime 2: Inflation is *negatively correlated* with aggregate consumption growth.

Under which of these two policy regime will the price of the bond be highest? And under which of these two policy regimes will the expected return on holding nominal bonds be highest? Briefly explain why.

3. In order to find the true value function and decision rule(s) we have studied two methods: (i) "guess and verify" which gives us a closed-form analytical solution and (ii) numerical techniques which gives us a numerical approximation.

   (a) Briefly explain why we are not using "guess and verify" for all recursive problems.

   (b) In class we have used a numerical technique called discrete value function iteration to find the value function and decision rule(s). Explain first what a decision rule is and then carefully explain how you would use the approximation of the true decsision rule to generate artificial data for our model economy.