The exam consists of three parts: A, B, and C. In the grading, part A is given 20% weight, part B is given 40% weight, and part C is given 40% weight.

Part A (20 %)

Suppose you worked as an advisor at the Norwegian central bank in February 2020 when the policy rate was 1.5%. Assume that the economy was affected by a pandemic in March 2020, similar to what happened. Assume also that you knew the full impact of the pandemic on the economy (similar to what we know today). Use what you have learned in this course about how the central bank should respond to shocks to provide a policy recommendation for how the central bank should adjust its interest rate.

Hint: In this problem, you are asked to consider only conventional interest rate policies.

Part B (40 %)

Assume that a household solves the following problem

$$\max_{c_1, c_2} \frac{c_1^{1-1/\gamma}}{1-1/\gamma} + \beta \frac{c_2^{1-1/\gamma}}{1-1/\gamma}$$

$$\text{subject to}$$

$$c_1 + \frac{c_2}{1+r} = y + \frac{y}{1+r}$$

where c is consumption, y is income, and r is the interest rate. $\beta \in [0,1]$ and $\gamma > 0$ are known parameters.

a) Show that the first-order condition can be written as

$$c_1^{-1/\gamma} = \beta(1+r)c_2^{-1/\gamma}. \tag{1}$$

Interpret equation (1).

b) Solve for consumption in period 1 as a function of income.

c) Compute the optimal consumption response in period 1 to an increase in the interest rate. Interpret in terms of substitution and income effects.
Note: You can get a part of the credits here if you can explain the substitution and income effects without being able to compute them.

The household problem in the New Keynesian model in lecture 2 can be written as (we ignore uncertainty here)

$$\max_{C_t, N_t} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-1/\gamma}}{1-1/\gamma} - \frac{N_t^{1+1/\phi}}{1+1/\phi} \right)$$

subject to

$$C_t + \frac{B_t}{1+r_t} \leq B_{t-1} + W_t N_t$$

and a transversality condition

where C is consumption, N is hours worked, B is bonds, W is the real wage, and r is the real interest rate. $\beta \in [0,1]$, $\gamma > 0$, and $\phi > 0$ are known parameters.

d) Explain why there is no income effect from interest rate changes in the household problem from lecture 2 if one starts from a steady state ($W_t = W$, $C_t = C$, $B_t = B$, $N_t = N$) with $B = 0$ and $\beta(1+r) = 1$.

Part C (40 %)

We are going to use the New Keynesian model to see how the economy responds to a demand shock under an interest rate rule. We assume that the model is

$$\pi_t = E_t\{\pi_{t+1}\} + \kappa x_t \tag{2}$$

$$x_t = E_t\{x_{t+1}\} - (i_t - E_t\{\pi_{t+1}\}) + d_t \tag{3}$$

$$i_t = \phi_\pi \pi_t \tag{4}$$

$$d_t = \rho d_{t-1} + \eta_t \tag{5}$$

$$\eta_t \sim N(0, \sigma_d) \tag{6}$$

$$\kappa = \left(1 + \frac{\phi + \alpha}{1-\alpha}\right)\left(\frac{1-\theta}{\theta}\right)(1-\theta)\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right) \tag{7}$$

where x is the output gap, π is inflation, i is the nominal interest rate, and d is a demand shock. κ, ϕ_π, ρ, σ_d, ϕ, α, θ, and ε are known parameters.

a) Insert (4) into (3) and guess that

$$
\begin{aligned}
x_t &= \psi_{xd} d_t \\
\pi_t &= \psi_{\pi d} d_t \\
E_t\{x_{t+1}\} &= \rho \psi_{xd} d_t \\
E_t\{\pi_{t+1}\} &= \rho \psi_{\pi d} d_t.
\end{aligned}
$$

Find the solutions for ψ_{xd} and $\psi_{\pi d}$.

b) Figure 1 shows the impulse responses of the output gap, inflation, the real interest rate, the nominal interest rate, and the demand shock to an initial shock to ϵ_0^d of -0.5. We have used the following calibration: $\phi = 5$, $\alpha = 0.25$, $\epsilon = 9$, $\theta = 0.75$, $\phi_\pi = 1.5$, and $\rho = 0.5$. Explain why the demand shock affects the economy the way it does in Figure 1.

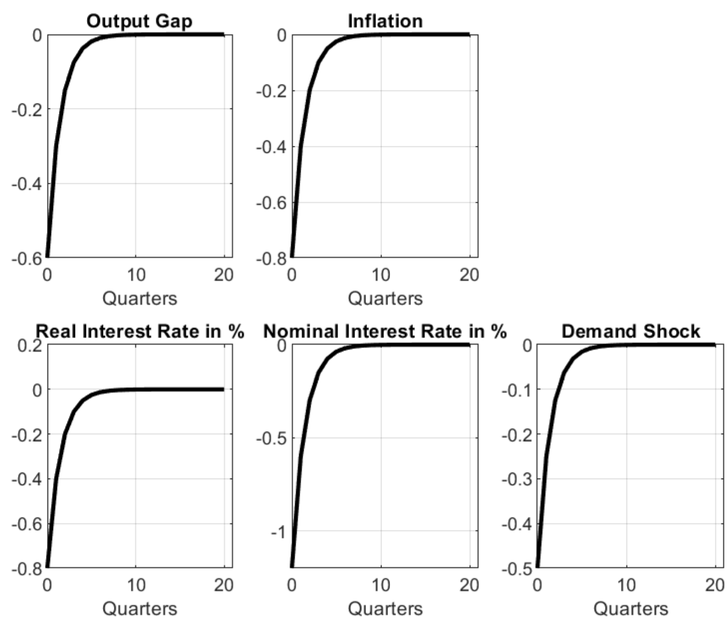

Figure 1: Impulse responses to a demand shock

c) We now change the calibration by increasing θ from 0.75 to 0.90. All other parameters are the same as in problem b. Figure 2 presents the impulse responses in the benchmark scenario and in the new calibration. Explain what θ is, how it changes the economic environment, and how agents alter their behavior. Next, explain why the aggregate responses of the economy change in the way presented in Figure 2.

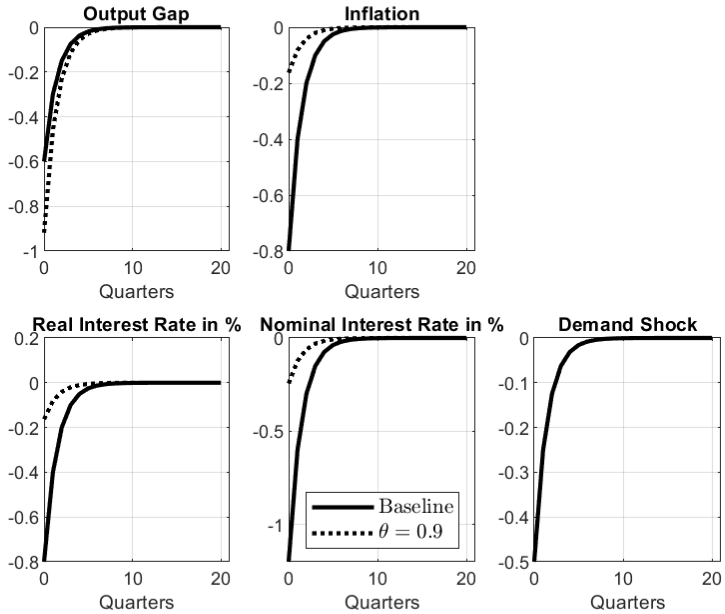

Figure 2: Impulse responses to a demand shock: variation in θ.

d) Assume now that the central bank follows optimal monetary policy instead of a Taylor rule (4) where the central bank cares about inflation and the output gap. The loss function is

$$L_t = \frac{1}{2} \sum_{k=0}^{\infty} \left(\pi_{t+k}^2 + \lambda x_{t+k}^2 \right)$$

where $\lambda > 0$ is a parameter.

Show that the solution for optimal monetary policy under discretion is

$$\lambda x_t = -\kappa \pi_t. \tag{8}$$

and interpret equation (8).

e) Show that the optimal solution for the interest rate is

$$i_t = d_t$$

and explain the intuition for the implied interest rate rule.

Solution proposal

PART A

A good answer should include

- a description of the situation in March 2020.
- a discussion of what kind of shock this is in terms of our model and how one should respond to such shocks. In particular, it should be discussed whether the type of shock is a demand, cost-push, or supply/technology shock and how the central bank should respond to each shock.
- a policy recommendation for how the central bank should adjust the interest rate.

PART B

a) Solve by standard method. This is the euler-equation. Marginal utility today is equal the discounted utility from saving today, getting the return, and consuming tomorrow.

b) Consumption in period 1 is

$$c_1 = \frac{y + \frac{y}{1+r}}{1 + \beta^\gamma (1+r)^{\gamma-1}}$$

c) The optimal consumption response in period 1 to a higher interest rate is

$$\begin{aligned}
\frac{\partial c_1}{\partial r} &= -\frac{y + \frac{y}{1+r}}{\left(1 + \beta^\gamma (1+r)^{\gamma-1}\right)^2} \beta^\gamma (1+r)^{\gamma-2} (\gamma - 1) - \frac{\frac{y}{(1+r)^2}}{1 + \beta^\gamma (1+r)^{\gamma-1}} \\
&= \underbrace{-\frac{y + \frac{y}{1+r}}{\left(1 + \beta^\gamma (1+r)^{\gamma-1}\right)^2} \beta^\gamma (1+r)^{\gamma-2} \gamma}_{\text{substitution effect}} + \underbrace{\frac{y}{\left(1 + \beta^\gamma (1+r)^{\gamma-1}\right)^2} \frac{1}{(1+r)^2} \left(\beta^\gamma (1+r)^\gamma - 1\right)}_{\text{income effect}}
\end{aligned}$$

The substitution effect is that with a higher interest rate, it is marginally more attractive to save such that the household reduces current consumption. The income effect is that a higher interest rate affects the household's lifetime resources. It is positive if the household is a saver and negative if it is a borrower.

d) There is no income effect in a steady state with $B = 0$ and $\beta(1+r)$ because then income is constant across time and since there is no saving and $\beta(1+r)$, then consumption is also constant across time. The household is therefore neither a saver not a borrower and it is therefore no income effect from interest rate changes.

PART C

a) Solve by the method of undetermined coefficients. The solution is

$$\psi_{xd} = (1-\rho)\Lambda_d \tag{9}$$

$$\psi_{\pi d} = \kappa\Lambda_d \tag{10}$$

$$\Lambda_d = \frac{1}{(1-\rho)^2 + \kappa(\phi_\pi - \rho)}. \tag{11}$$

b) A negative demand shock reduces output for a given level of inflation and interest rate. This lowers inflation through the Phillips curve. The central bank responds to the fall in inflation by lowering the interest rate and this reduction is sufficiently large to reduce the real interest rate, thus dampening the initial impact somewhat. The total effect is that the output gap is lower, inflation is reduces, the nominal interest rate falls, and the real interest is lower.

c) θ is the parameter for the probability of being stuck with the price in the next period. A higher θ has two effects on this economy. First, prices are less flexible because less firms will respond to any change in current aggregate demand (the output gap). Second, prices are also less responsive to changes in current demand because with a relatively higher θ, the firms care less about the current period relative to future period. For both reasons, κ is lower. This means that the initial shock has less effect on inflation. Since the central bank cares only about inflation, the interest rate response is also weaker. The initial drop in the output gap is then allowed to a greater extent to affect the economy since it has less impact on inflation. The total effect of a higher θ is therefore that the inflation, nominal interest rate, and real interest responses are dampened, while the output gap response is somewhat stronger.

d) Solve by standard method. The interpretation is that the marginal cost of reducing output (λx_t) must be weighted against the marginal gains from reducing inflation (-$\kappa\pi_t$).

e) For a demand shock, there is no trade-off. The optimal solution for the central bank is $\lambda x_t = -\kappa \pi_t = 0$. To achieve that solution, one just sets the interest rate to cancel out the demand shock, i.e., $i_t = d_t$.