The exam consists of three parts, A, B, and C. In the grading, Part A has 20% weight, Part B has 30% weight, and Part C has 50% weight. Remember to allocate your time accordingly.

Part A (20 %)

This part contains two problems. You need to answer all to get full score. Your answers to each problem should not exceed one page.

a) **Balance sheet policies.** Explain how the central bank can use its balance sheet as a monetary policy tool to stimulate the economy. How effective is this type of policy?

b) **Log-linearization.** Log linearize the following equation around k and z.

$$k_{t+1} = sz_t k_t^{\alpha} + (1 - \delta)k_t.$$

Part B (30%)

This part contains three problems. You need to answer all to get full score.

The household problem in the standard New-Keynesian model is

$$\max_{C_t,N_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right)$$

subject to

$$P_t C_t + \frac{B_t}{1+i_t} \leq B_{t-1} + W_t N_t + D_t$$

$$\lim_{T\to\infty} E_t \left\{ \beta^{T-t} \frac{U_{C,T}}{U_{C,t}} \frac{B_T}{P_T} \right\} \geq 0$$

where C is consumption, N is labor supply, B is a nominal bond, W is the nominal wage, D is dividends, P is the price of consumer goods, and i is the nominal interest rate.

a) **The Euler equation.** Solve the household problem and find the expression for the intertemporal consumption Euler equation. Explain the intuition. What trade-off does the household face? Which factors influence this decision?

b) **Consumption-labor.** Solve the household problem and find the expression for the intratemporal consumption-labor decision. Explain the intuition. What trade-off does the household face? Which factors influence this decision?

c) **GHH preferences.** Assume that the utility function instead is

$$U(C_t, N_t) = \log\left(C_t - \frac{N_t^{1+\phi}}{1+\phi} \right).$$

Solve for the *intratemporal consumption-labor choice*. Explain how the solution is different from b). Which channel is absent from the solution in c) compared to the solution in b).

Part B (50 %)

This part contains five problems. You need to answer all to get full score.

We are going to use the New Keynesian model to see how the economy responds to a discount rate shock under an interest rate rule. Assume that the model is

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t + u_t \tag{1}$$

$$y_t = E_t\{y_{t+1}\} - (i_t - E_t\{\pi_{t+1}\}) \tag{2}$$

$$i_t = \phi_\pi \pi_t \tag{3}$$

$$u_t = \rho_u u_{t-1} + \nu_t^u, \qquad \nu_t^u \sim N(0, \sigma_u) \tag{4}$$

$$\kappa = \left(1 + \frac{\phi + \alpha}{1 - \alpha}\right)\left(\frac{1 - \theta}{\theta}\right)(1 - \beta\theta)\left(\frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon}\right) \tag{5}$$

where π is inflation, y is the output gap, and u is a cost-push shock. β, κ, ϕ_π, ρ_u, σ_u, ϕ, α, θ, and ε are parameters of the model.

a) **Guess and verify.** Guess that

$$x_t = \psi_{xu} u_t$$
$$\pi_t = \psi_{\pi u} u_t$$
$$E_t\{x_{t+1}\} = \rho \psi_{xu} u_t$$
$$E_t\{\pi_{t+1}\} = \rho \psi_{\pi u} u_t.$$

Find the solutions for ψ_{xu} and $\psi_{\pi u}$.

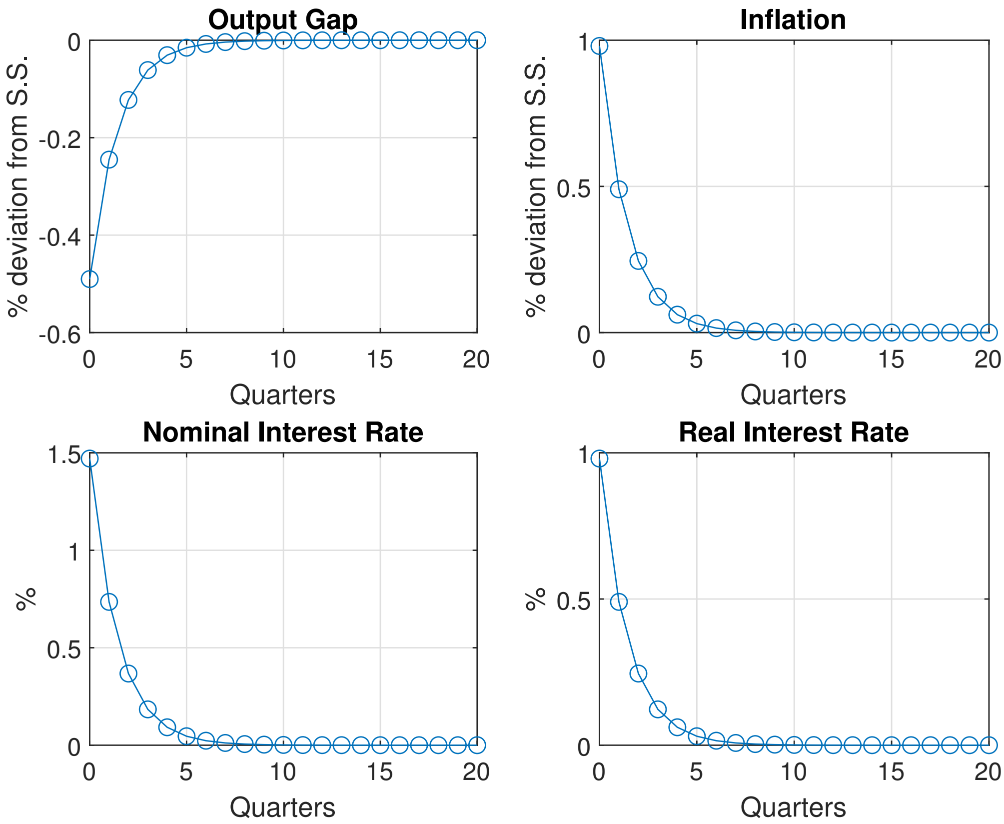

Figure 1: Impulse Responses

b) **Impulse responses.** Figure 1 shows the impulse responses of the output gap, inflation, the nominal interest rate, and the real interest rate to a positive cost-push shock. The calibration is as follows: $\beta = 0.99$, $\phi = 5$, $\alpha = 0.25$, $\varepsilon = 9$, $\theta = 0.75$, $\phi_\pi = 1.5$, and $\rho_u = 0.5$. Explain in words how the cost-push shock affects the economy and how the central bank responds to it.

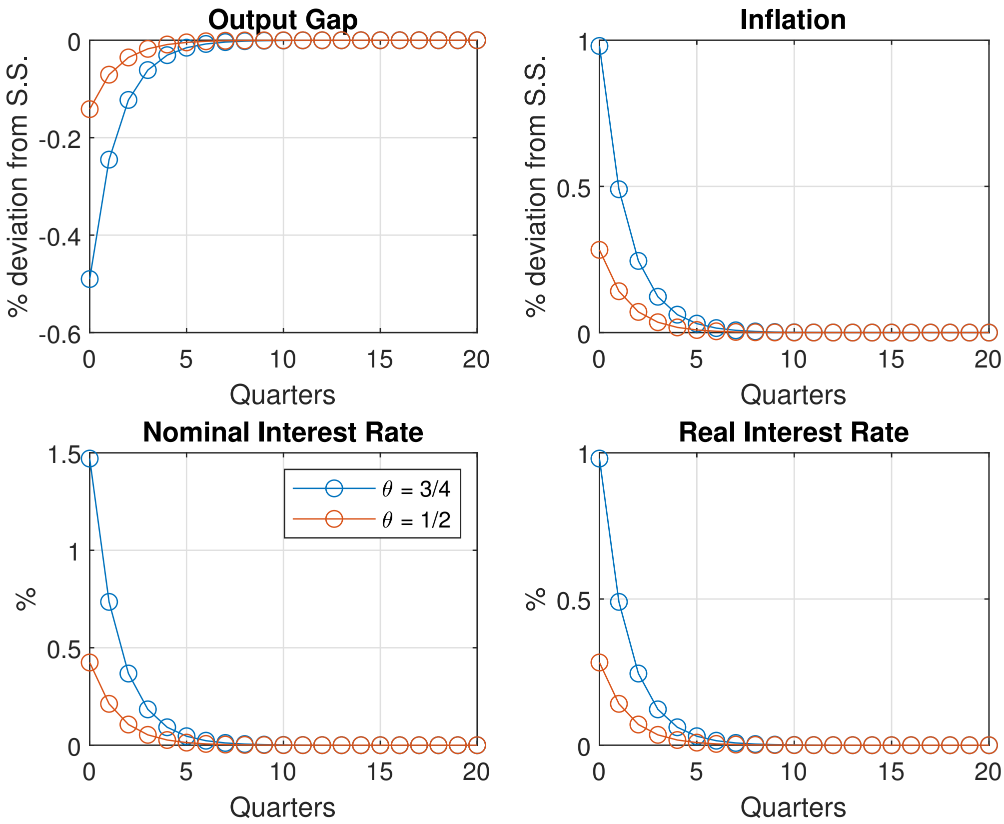

Figure 2: Impulse Responses

c) **Parameter changes.** We now reduce θ from 0.75 to 0.50. All other parameters are the same as in problem b). Figure 2 presents the impulse responses in the benchmark scenario and in the new calibration. Explain what θ is, how it changes the economic environment, and explain how agents alter their behavior. Next, explain why the aggregate responses of the economy change in the way presented in Figure 2.

d) **Optimal monetary policy.** We are now going to solve for optimal monetary policy under commitment (unconstrained optimum). Assume that the central bank minimizes the following loss function

$$\mathcal{L} = \frac{1}{2}\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k (\lambda y_{t+k}^2 + \pi_{t+k}^2) \tag{6}$$

where $\lambda > 0$. Minimize the loss function (6) subject to the Phillips curve (1) under commitment. What is the trade-off the central bank faces?

Solution Proposal

Part A

a) A good answer describes

- Distinction between pure quantitative easing (QE) and credit easing (CE).
- Gives example of central banks which have implemented QE and CE
- Outlines how QE and CE should provide stimulus (no need for equations, but the the intuition behind the various channels should be explained)
  - QE: Money multiplier and velocity of money
  - CE: Portfolio rebalancing and bank lending channel

b) The solution is

$$k\tilde{k}_{t+1} \approx [\alpha szk^{\alpha-1} + (1-\delta)]k\tilde{k}_t + [sk^{\alpha}]z\tilde{z}_t$$

where ˜ denotes log-linear variables.

Part B

a) The intertemporal consumption Euler-equation

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left\{C_{t+1}^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right\}.$$

The marginal utility of consuming today is equal to the (discounted) marginal utility of consuming the gross real returns from saving the same unit today.

b) The intratemporal consumption-labor choice

$$C_t^{-\sigma}\frac{W_t}{P_t} = N_t^{\phi}.$$

The household can choose between work or leisure. If it works one more unit, it makes W/P real units of consumption goods, which gives marginal utility equal to $C_t^{-\sigma}$. The marginal cost is that it dislikes working and the marginal disutility from working is N_t^{ϕ}.

The expression can also be explained in terms of wealth and substitution effects on labor supply. It is then instructive to rewrite the equation as $n_t = \frac{1}{\phi}(w_t - p_t) - \frac{\sigma}{\phi}c_t$ where small letters denote logs. Then the first term with $w_t - p_t$ describes the substitution effect (higher wages → higher labor supply) while the second term describes the wealth effect (higher wages → more consumption → lower marginal utility → need more (and more) compensation to work more).

c) The new intratemporal consumption-labor choice

$$\frac{W_t}{P_t} = N_t^{\phi}$$

The is now no wealth effect on labor supply and labor supply is determined only by the substitution effect ($n_t = \frac{1}{\phi}(w_t - p_t)$).

Part C

a) Solve by method of undetermined coefficients. The solutions are

$$\psi_{\pi u} = \frac{1 - \rho_u}{(1 - \beta\rho_u)(1 - \rho_u) + \kappa(\phi_\pi - \rho_u)}$$
$$\psi_{xu} = -\frac{\phi_\pi - \rho_u}{(1 - \beta\rho_u)(1 - \rho_u) + \kappa(\phi_\pi - \rho_u)}$$

b) Initially, the cost push shock raises prices. The central bank responds to this price rise by increasing the interest rate more than one-for-one. Hence, the real interest rate increases and output contracts. As output contracts, the initial impact on inflation is somewhat dampened. The total effect is that inflation goes up, output declines, the nominal interest rate goes up, and the real interest rate goes up.

c) θ is the parameter for the probability of being stuck with the price in the next period. A lower θ has two effects on this economy. First, prices are more flexible because more firms will respond to any change in current aggregate demand (the output gap). Second, prices are also more responsive to changes in current demand because with a relatively lower θ, the firms care more about the current period relative to future period. For both reasons, κ is higher. Since the central bank initially responds to the cost-push shock by increasing the real interest rate, this reduces the output gap. With a higher κ, this change in the output gap has a greater effect on inflation than before. Inflation therefore increases by less than in the baseline calibration. Further, since inflation increases by less, the nominal interest rate also has to increase by less, dampening the effect also on output. The total effect of a lower θ and therefore higher κ is that all responses are dampened.

d) The first-order conditions are

$$
\begin{aligned}
x_{t+k} - x_{t+k-1} &= -\frac{\kappa}{\lambda}\pi_{t+k} && \text{for } k > 0 \\
x_t &= -\frac{\kappa}{\lambda}\pi_t && \text{for } k = 0
\end{aligned}
$$

By a slight rewrite, one can explain the intuition for the trade-off as

$$
\begin{aligned}
x_{t+1} = x_t - \frac{\kappa}{\lambda}\pi_{t+1} &= -\frac{\kappa}{\lambda}\pi_t - \frac{\kappa}{\lambda}\pi_{t+1} \\
\underbrace{\beta\lambda x_{t+1}}_{\text{Discounted marginal cost}} &= \underbrace{-\beta\kappa(\pi_{t+1} + \pi_t)}_{\text{Discounted marginal benefit: reducing inflation in both periods}}
\end{aligned}
$$

where the gains from commitment comes because any change in output in the future also affects inflation in all periods from now until that future period.