ECON 4325 – Monetary Policy and Business Fluctuations

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- A simple model of a classical monetary economy.
  - Perfect competition and flexible prices in all markets.
- A useful benchmark for later analysis, but many of the predictions are at odds with the empirical evidence.

- Households:
  - Complete financial markets.
  - Perfectly competitive labor market.
- Firms:
  - Competitive firms (monopolistic competition and sticky prices later).
  - Cobb-Douglas production function with labour as the only input.
- General equilibrium – a so-called DSGE-model (dynamic, stochastic, general equilibrium).

- The representative household chooses labor supply, consumption, and one-period bonds. Maximize discounted expected utility:

$$E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, N_{t+k}\right), \tag{1}$$

where β is the discount factor.

  - U – period utility
  - C and N – consumption and employment ($1-$ leisure).

- We use the following period utility function:

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{2}$$

where $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution and $\frac{1}{\varphi}$ is the Frisch labor supply elasticity.

  - $\frac{1}{\sigma}$ measures how willing the household is to substitute consumption over time when the real interest rate changes.
  - $\frac{1}{\sigma}$ measures how labor supply increases when the real wage increases.

- Sequence of budget constraints:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t, \tag{3}$$

  - P – price of consumption goods.
  - W and T – nominal wages and taxes (net of dividends from ownership of firms).
  - Q and B – price and quantity of one-period risk-free nominal bonds that pay one nominal unit on maturity.

- Solvency constraint.

- Let us look Lagrangian:

$$\begin{aligned} L_t &= E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[\frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi} - \Lambda_{t+k} \left[P_{t+k} C_{t+k} \right.\right.\right. \\ &\quad \left.\left.\left. + \, Q_{t+k} B_{t+k} - B_{t+k-1} - W_{t+k} N_{t+k} + T_{t+k} \right] \right] \right\} \end{aligned} \tag{4}$$

- First-order conditions:

$$\begin{aligned} \frac{\partial L_t}{\partial C_t} &= C_t^{-\sigma} - \Lambda_t P_t = 0 \Rightarrow \Lambda_t = \frac{C_t^{-\sigma}}{P_t}, \\ \frac{\partial L_t}{\partial N_t} &= -N_t^{\varphi} + \Lambda_t W_t = 0 \Rightarrow \Lambda_t = \frac{N_t^{\varphi}}{W_t}, \\ \frac{\partial L_t}{\partial B_t} &= -\Lambda_t Q_t + \beta E_t \left\{ \Lambda_{t+1} \right\} = 0 \Rightarrow Q_t = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \right\}. \end{aligned}$$

- We then get:

$$C_t^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \equiv \Omega_t, \tag{5}$$

where Ω_t is period t real wage.

- The optimality condition for labor supply:
  - The real wage increases with hours worked (compensating for increases in marginal disutility).
  - Increases with consumption (which makes the "utility value" of the real wage lower).

- Moreover, we have

$$\beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \Pi_{t+1}^{-1} \right\} = Q_t, \tag{6}$$

where $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross inflation (and $\pi_t \equiv \log \Pi_t$ is the rate of inflation).

- The consumption Euler-equation:
  - Reflects the household's preferences for consumption smoothing.

- Note that the nominal (risk-free) interest rate is defined as

$$i_t \equiv -\log Q_t,$$

since Q_t is the period t price of getting one monetary unit in all states in period $t+1$.

  - The bond yield is implicitely given by $Q_t = (1 + yield)^{-1}$.
  - We therefore have $i = -\log Q_t = \log (1 + yield) \simeq yield$.

- Consider the following two-period model:

$$\max_{C_t, C_{t+1}} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} + \beta \frac{C_{t+1}^{1-\sigma}}{1-\sigma} \right\}$$

- Let the household's budget constraints be:

$$\begin{aligned} P_t C_t &= \Omega - S_t, \\ P_{t+1} C_{t+1} &= (1 + i_t) S_t, \end{aligned}$$

where S_t is period t saving.

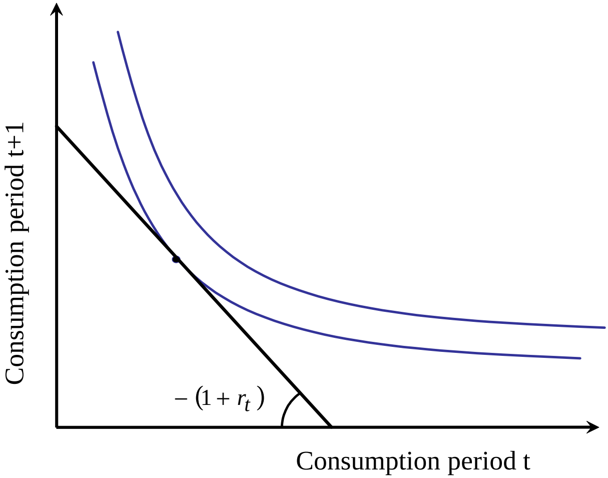
- Household's problem

$$\max_{S_t} \left\{ \frac{\left(\left(\Omega - S_t\right)/P_t\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(S_t\left(1+i_t\right)/P_{t+1}\right)^{1-\sigma}}{1-\sigma} \right\}$$

- FOC:

$$\begin{aligned} -C_t^{-\sigma}/P_t + \beta C_{t+1}^{-\sigma}\left(1+i_t\right)/P_{t+1} &= 0, \\ \beta\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\Pi_{t+1}^{-1}\right] &= \frac{1}{1+i_t} \Leftrightarrow \\ -\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} &= -\frac{1}{1+r_t}, \end{aligned}$$

where r_t is the real interest rate.


Consumption period t+1
− (1 + r t)
Consumption period t

- There is a representative firm that has access to the following production technology:

$$Y_t = A_t N_t^{1-\alpha}, \tag{7}$$

where Y_t and N_t are production and labor input, and $\log A_t \equiv a_t = \rho_a a_{t-1} + \varepsilon_t^a$.

- Price-taker in all markets (the labor market and the goods market).

- The firm maximizes profits:

$$\max_{Y_t, N_t} \left[P_t Y_t - W_t N_t \right], \tag{8}$$

s.t.

$$Y_t = A_t N_t^{1-\alpha} \tag{9}$$

- First-order condition:

$$MPL_t \equiv (1-\alpha) \frac{Y_t}{N_t} = \Omega_t. \tag{10}$$

- All markets clear:

$$Y_t = C_t, \tag{11}$$
$$N_t^s = N_t^d = N_t. \tag{12}$$

- In addition $B_t = 0$ (zero net savings).

- Labor supply:
  - We start by looking at the steady state:

$$C^{\sigma}N^{\varphi} = \Omega,$$

  - Then we log-linearizing labor supply:

$$\begin{aligned} e^{\sigma c_t + \varphi n_t} &= e^{\omega_t}, \\ e^0 + e^0 \sigma (c_t - 0) + e^0 \varphi (n_t - 0) &= e^0 + e^0 (\omega_t - 0) \\ \varphi n_t + \sigma c_t &= \omega_t \end{aligned}$$

  and $\frac{1}{\varphi}$ measures how much labor supply increases when the real wage increases.

- Consumption Euler-equation:

$$1 = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{Q_t \Pi_{t+1}} \right\}.$$

  - Steady state

$$\begin{aligned} Q\Pi &= \beta \left(\frac{C}{C} \right)^{-\sigma} = \beta \\ \Rightarrow & -i + \pi = \log \beta \equiv -\rho, \end{aligned}$$

  where ρ is household's discount rate. This implies a steady state real rate, $r \equiv i - \pi = \rho$.

- Consumption Euler-equation (cont'd):
    - Log-linearizing:

$$
\begin{aligned}
1 &= E_t \left\{ e^{-\sigma(c_{t+1}-c_t)} e^{-\log Q_t - \log \Pi_{t+1} + \log \beta} \right\} \\
&= E_t \left\{ e^{-\sigma(c_{t+1}-c_t) + (i_t - \pi_{t+1} - \rho)} \right\} \\
1 &= e^0 - e^0 E_t \left\{ \sigma \left(c_{t+1} - c_t \right) - \left(\left(i_t - i \right) - \left(\pi_{t+1} - \pi \right) \right) \right\} \\
c_t &= E_t c_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - \rho \right).
\end{aligned}
$$

- The consumption Euler-equation:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(r_t - \rho), \tag{13}$$

where we have used $r_t \equiv i_t - E_t \pi_{t+1}$

- Consumption smoothing: parameter $\frac{1}{\sigma}$ measures by how much consumption increases when the real interest rate drops.
- Consumption is a pure forward-looking – or jump – variable.

- Solving the equation forward gives:

$$c_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\left(r_{t+k}-\rho\right) = -\frac{1}{\sigma}\left(r_t^L-\rho\right), \tag{14}$$

  where r^L is related to long real rates.

- Given the expectation hypothesis the relationship between short and long (real) rates with maturity T, r_t^T, is given by:

$$r_t^T = \rho + \frac{1}{T}\sum_{k=0}^{T}\left(r_{t+k}-\rho\right).$$

- Therefore:

$$r_t^L \approx T r_t^T$$

Firms

- Output

$$y_t = a_t + (1 - \alpha)\, n_t, \tag{15}$$

- Labor demand

$$\omega_t = y_t - n_t, \tag{16}$$

and we also have productivity $a_t = \rho_a a_{t-1} + \varepsilon_t^a$.

Market clearing

- Consumption equals output:

$$y_t = c_t. \tag{17}$$

Money demand and monetary policy

- We assume that (log-linearized) money demand is given by

$$m_t - p_t = y_t - \eta i_t, \tag{18}$$

  where η is the semi interest rate elasticity.

- Let monetary policy be given by:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m. \tag{19}$$

- Let us solve the model, i.e. show how the endogenous variables depend the exogenous variables.
- To this end, let us repeat the complete set of equations

$$
\begin{array}{l|l}
c_t = E_t c_{t+1} - \frac{1}{\sigma}\left(r_t - \rho\right), & r_t = i_t - E_t \pi_{t+1}, \\
\omega_t = \phi n_t + \sigma c_t, & w_t - p_t = \omega_t, \\
y_t = n_t + a_t, & m_t - p_t = y_t - \eta i_t, \\
\omega_t = y_t - n_t, & \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m. \\
y_t = c_t, & \\
a_t = \rho_a a_{t-1} + \varepsilon_t^a &
\end{array}
$$

- "Classical Dichotomy": Real variables determined independently of monetary policy (neutrality)
- Optimal policy: undetermined.
- Specification of monetary policy needed to determine nominal variables.

- Let us first solve for the monetary part of the model (and we now assume that $a_t = 0$).
- Substitute for the nominal interest rate in the money demand function using the Fisher equation:

$$m_t - p_t = 0 - \eta E_t \{ p_{t+1} - p_t \},$$

which can be written as

$$p_t = \frac{\eta}{1+\eta} E_t p_{t+1} + \frac{1}{1+\eta} m_t.$$

- This can be solved forward to yield:

$$p_t = \frac{1}{1+\eta} E_t \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t m_{t+k}.$$

- We want to rewrite the relationship this in terms of changes in nominal money:

$$
\begin{aligned}
p_t &= m_t - m_t + \frac{1}{1+\eta} m_t + \frac{\eta}{1+\eta} E_t m_{t+1} + \\
&\quad - \frac{\eta}{1+\eta} E_t m_{t+1} + \frac{1}{1+\eta} \frac{\eta}{1+\eta} E_t m_{t+1} + \left(\frac{\eta}{1+\eta} \right)^2 E_t m_{t+2} \\
&\quad + \ldots
\end{aligned}
$$

- We can then write:

$$
\begin{aligned}
p_t &= m_t - \frac{\eta}{1+\eta} m_t + \frac{\eta}{1+\eta} E_t m_{t+1} + \\
&\quad + \frac{\eta}{1+\eta} \left(\frac{1}{1+\eta} - 1 \right) E_t m_{t+1} + \left(\frac{\eta}{1+\eta} \right)^2 E_t m_{t+2} \\
&\quad + \ldots
\end{aligned}
$$

- The price level is therefore:
- We can then write:

$$
\begin{aligned}
p_t &= m_t + \frac{1}{1+\eta} E_t \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \Delta m_{t+k} \\
&= m_t + \frac{\eta \rho_m}{1+\eta \left(1-\rho_m\right)} \Delta m_t
\end{aligned}
$$

- If $\rho_m > 0$ (the parameter is often calibrated to to 0.5 based on empirical evidence), the price level should respond more than one-for-one with the increase in the money supply.
- This prediction is in stark contrast to the sluggish response of the price level observed in empirical estimates of the effects of monetary policy shocks.

- We can solve the (real side of the) model explicitly as follows:
  - Use the labor supply and demand equations and combine it with the aggregate resource constraint:

$$y_t - n_t = \varphi n_t + \sigma y_t,$$

  - Next, combine the above equation with the production function

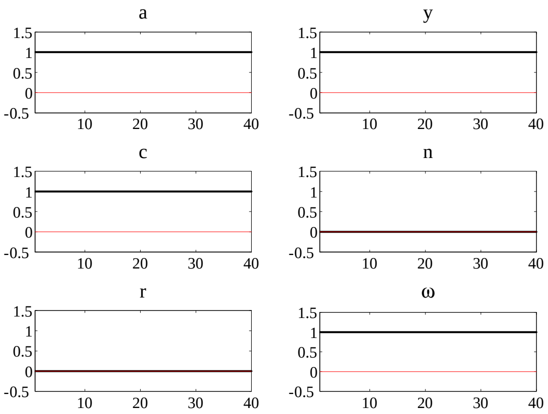
$$\begin{aligned} (1-\sigma)\, y_t &= \frac{(1+\varphi)}{1-\alpha}\,(y_t - a_t) \\ y_t &= \psi_{ya} a_t \end{aligned}$$

    which only depend on productivity and $\psi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

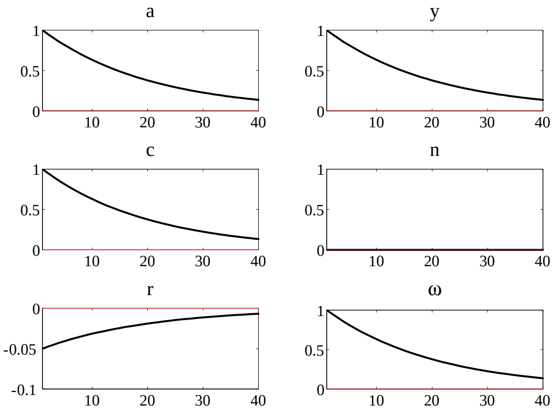
- The solution to the (real side of the) model is then:

$$
\begin{aligned}
y_t &= \psi_{ya} a_t \text{ where } \psi_{ya} = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \left.\right\} \begin{matrix} >1 \text{ if } \sigma<1 \\ =1 \text{ if } \sigma=1 \\ <1 \text{ if } \sigma>1 \end{matrix} \\
n_t &= \psi_{na} a_t \text{ where } \psi_{na} = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} \left.\right\} \begin{matrix} >0 \text{ if } \sigma<1 \\ =0 \text{ if } \sigma=1 \\ <0 \text{ if } \sigma>1 \end{matrix} \\
\omega_t &= \psi_{\omega a} a_t \text{ where } \psi_{na} = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \\
r_t - \rho &= \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} = -(1-\rho_a)\sigma\psi_{ya} a_t
\end{aligned}
$$

Parameters: $\sigma = 1$, $\beta = 0.99$, $\alpha = 0$, $\varphi = 1$



Parameters: $\sigma = 1$, $\beta = 0.99$, $\alpha = 0$, $\varphi = 1$, $\rho_a = 0.95$



- So far, money has only served as a unit of account (often referred to as cashless economies).
- We now assume that money generate utility:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) \tag{20}$$

- The budget constraint becomes:

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t. \tag{21}$$

- The optimality conditions are:

$$-\frac{U_{N,t}}{U_{C,t}} = \Omega_t, \tag{22}$$

$$\beta E_t \left\{ \frac{U_{C,t+1}}{U_{C,t+1}} \Pi_{t+1}^{-1} \right\} = Q_t \tag{23}$$

$$\frac{U_{N,t}}{U_{C,t}} = 1 - Q_t \simeq \frac{i_t}{1+i_t} \tag{24}$$

- Interpretation of the latter:
    - LHS: increased utility from holding more money (in consumption units).
    - RHS: alternative cost (one monetary unit, minus the cost of buying a bond that gives one monetary unit in period $t+1$, as is the case when holding money).

- Two cases:
  1. Utility is seperable in real balances: neutrality.
  2. Utility is non-seperable in real balances: non-neutrality.
- Even in the case of non-neutrality there are very small real effects from monetary shocks.
- Optimal policy: Friedman rule (zero nominal interest rate and $\pi = -\rho$).