ECON 4325 – Monetary Policy and Business Fluctuations

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The Canonical New-Keynesian Model

- Households:
  - Complete financial markets. Perfectly competitive labor market.
- Monopolistically competitive firms:
  - Production function with labour as the only input.
  - Firms set their price. Prices are sticky.
- General equilibrium – a so-called DSGE-model (dynamic, stochastic, general equilibrium).

- A representative household maximizes:

$$E_t \sum_{k=0}^{\infty} \beta^k U\left(C_{t+k}, N_{t+k}\right), \tag{1}$$

where $C_t \equiv \left(\int_0^1 C_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ and subject to a sequence of budget constraints:

$$\int_0^1 P_t\left(i\right) C_t\left(i\right) di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \tag{2}$$

- Firms choose prices, output and labor input to maximize:

$$\max \sum_{k=0}^{\infty} E_t \left\{ Q_{t,t+k} \left[P_{t+k}(i)\, Y_{t+k}(i) - W_{t+k} N_{t+k}(i) \right] \right\}, \qquad (3)$$

subject to a sequence of constraints

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \qquad (4)$$

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \qquad (5)$$

$$P_{t+1}(i) = \begin{cases} P^*_{t+1}(i) & \text{with prob. } (1-\theta) \\ P_t(i) & \text{with prob. } \theta. \end{cases} \qquad (6)$$

- Social planner maximizes (representative) household's welfare:

$$\max U\left(C_t, N_t\right) \tag{7}$$

subject to:

$$C_t = \left(\int_0^1 C_t\left(i\right)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

$$C_t\left(i\right) \leq A_t N_t\left(i\right)^{1-\alpha} \quad \forall i \in [0,1] \tag{9}$$

$$N_t = \int_0^1 N_t\left(i\right) di. \tag{10}$$

- The social planner problem implies the following optimality conditions:

$$C_t(i) = C_t \;\forall i \in [0,1] \qquad (11)$$

$$N_t(i) = N_t \;\forall i \in [0,1] \qquad (12)$$

  - Consume the same amount of every good, which implies that the use of labor is equal across firms.

- In addition we have:

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = (1-\alpha)\frac{Y_t}{N_t}. \qquad (13)$$

  - $MRS = MRT$
  - LHS: Marginal cost (in units of consumption goods) of increasing the use of labor in production.
  - RHS: Marginal increase in production of increasing the use of labor in production.

- Let us assume that the fiscal authorities pay an employment subsidy τ_w to firms per unit of labor. In that case their first-order condition becomes:

$$P_t = \mu \frac{(1-\tau_w) W_t}{(1-\alpha) Y_t / N_t}, \tag{14}$$

where $\mu = \frac{\epsilon}{\epsilon - 1}$.

- Use the labor supply equation to get rid of the real wage:

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = \frac{1-\alpha}{\mu(1-\tau_w)} \frac{Y_t}{N_t}. \tag{15}$$

- We therefore get the efficient allocation if $\tau_w = 1 - \frac{1}{\mu}$.

- In the sticky-price model there are two sources of inefficiencies:
  1. Fluctuations in the mark-up over marginal costs. Let us define the mark-up as:

$$\mu_t = \frac{P_t}{\frac{(1-\tau_w) W_t}{(1-\alpha) Y_t / N_t}} = \mu \frac{(1-\alpha) Y_t / N_t}{-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)}},$$

where we have assumed an optimal employment subsidy. Rewriting gives:

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = \frac{\mu}{\mu_t} (1-\alpha) Y_t / N_t. \tag{16}$$

- In the sticky-price model there are two sources of inefficiencies:
  2. Due to staggered price setting (not all firms change their price in a given period), we will have $P_t(i) \neq P_t(j)$ for any pair of goods (i, j) whose prices are not adjusted in the same period. From the demand function for goods we can write $\frac{C_t(i)}{C_t(j)} = \left(\frac{P_t(i)}{P_t(j)}\right)^{-\epsilon}$, and therefore different quantities are produced and consumed; and, as a result, $N_t(i) \neq N_t(j)$.

- We can do a second order approximation to household's welfare in the case of an employment subsidy (for details see the appendix):

$$\mathbb{W}_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_{t+k}^2 + \frac{\epsilon}{\lambda} \pi_{t+k}^2 \right], \qquad (17)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

  - The loss function is increasing in the variance of the output gap and inflation. The former is due to the variability of mark-ups; the second is due to cost of price dispersion.

- The loss function:

$$\mathbb{W}_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_{t+k}^2 + \frac{\epsilon}{\lambda} \pi_{t+k}^2 \right]. \tag{11}$$

  - Fluctuations in the output gap is more costly if:
    - σ is high (high "risk-aversion")
    - φ is high (labor supply elasticity is low)
    - α is high (more decreasing returns to scale in production).
  - Fluctuations in inflation is more costly if:
    - ϵ is high (substitution between goods is high)
    - λ is low (more price stickiness)

- The loss function is:

$$\mathbb{W}_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_{t+k}^2 + \frac{\epsilon}{\lambda} \pi_{t+k}^2 \right], \tag{11}$$

- The model economy can be written as:

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^n \right), \tag{18}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \, \widetilde{y}_t, \tag{19}$$

where

$$r_t^n \equiv \rho + \sigma E_t \Delta y_{t+1}^n,$$

and $\kappa \equiv \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$.

- What is optimal monetary policy in the our simple model?

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \frac{1}{\sigma} \left(i_t - E_t \pi_{t+1} - r_t^n \right), \quad (20)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \, \widetilde{y}_t, \quad (21)$$

- The divine coincidence:
    - Set interest rates such that $\widetilde{y}_t = 0$ for all t.
    - This implies that $\pi_t = 0$.
- The nominal interest rate must be such that $r_t = i_t - E_t \pi_{t+1} = r_t^n$.

- What is wrong with the following rule:

$$i_t = r_t^n \tag{22}$$

- The rule is consistent with optimal policy:
  - Inflation and output gap are zero. The nominal and real interest rate equals the natural real rate.
- The rule is also consistent with many other outcomes (multiple equilibria).

The Taylor principle: "Adjust the nominal interest rate more than one-for-one with changes in inflation"

- What happens if the central bank does not follow the Taylor principle?
  - Suppose households increase consumption, without any change in economic fundamentals. Production increases and thereby the marginal cost. Inflation increases.
  - The CB does not change nominal interest rates (because r_t^n is constant). The real interest rate decreases.
  - The reduction in real interest rates justifies the higher consumption.

- If the CB increase nominal rates by more than one-for-one the real interest rate increases.
- The Taylor principle is important in the design of monetary policy rules. Avoids that the central bank becomes a source of unnecessary fluctuations in economic activity.
- Clarida, Galí, and Gertler (2000) and Lubik and Schorfeide (2004): change from passive to active monetary policy in the early 1980's can explain the observed stabilization of macroeconomic outcomes in the US.

- Consider the following rule:

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t, \tag{23}$$

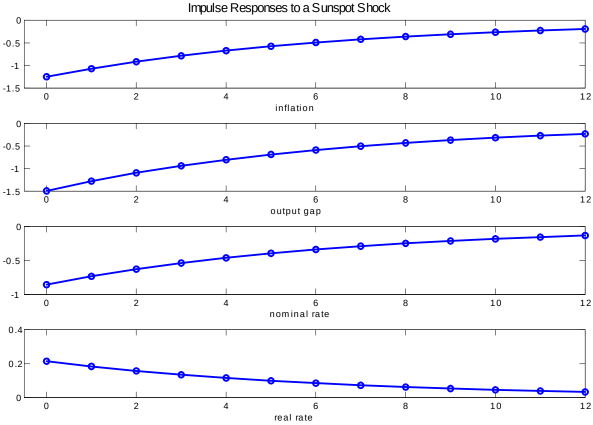
which is a Taylor-type rule.

- John Taylor (1993) argues that this rules tracks monetary policy under Greenspan-Volcker.

- Let us consider the following parameter values:

α	φ	σ	ϵ	β	θ	λ	κ
1/3	1	1	6	0.99	2/3	0.0425	0.1275

- We let the parameters in the policy rule be $\phi_\pi = 0.8$ and $\phi_y = 0$.
- We plot impulse responses to a sunspot shock as in Clarida, Galí and Gertler (2000).



See Galí (1997), “Solving Linear Dynamic Models with Sunspot Equilibria: A note”

- Let consider what would be the eventual implication of the rule above and the Phillips curve if inflation permanently increases by $d\pi$ (and assuming that the natural rate does not change):

$$
\begin{aligned}
di &= \phi_\pi d\pi + \phi_y d\widetilde{y}, && (24)\\
d\pi &= \beta d\pi + \kappa d\widetilde{y}, && (25)
\end{aligned}
$$

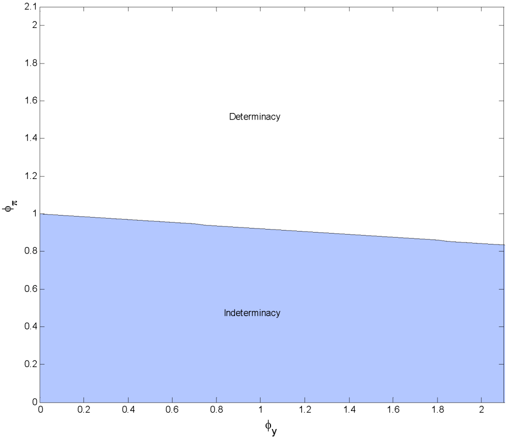
- Combining implies:

$$
di = \left[\phi_\pi + \phi_y \frac{(1-\beta)}{\kappa}\right] d\pi, \qquad (26)
$$

which implies that the real interest rate increases if $\kappa\left(\phi_\pi - 1\right) + \phi_y\left(1-\beta\right) > 0$.

  - If $\phi_y = 0$ then $\phi_\pi > 1$ is necessary (and sufficient) for determinacy.
  - If $\phi_y > 0$ then $\phi_\pi < 1$ might be sufficient.

Figure 4.1



Source: Galí (2008)

- The following simple rule therefore implements optimal policy:

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t. \tag{27}$$

- π_t and $\widetilde{y}_t$ are both zero for $i_t = r_t^n$. The two last terms are therefore both zero.
- The central bank only use the two terms as a "threat" of a strong response to an eventual deviation of the output gap and inflation from target.

- Often we see rules specified as follows:

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\left[r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t\right]. \tag{28}$$

- The condition is now still:

$$\kappa\left(\phi_\pi - 1\right) + \phi_y\left(1-\beta\right) > 0.$$

- The real interest rate must eventually increase!

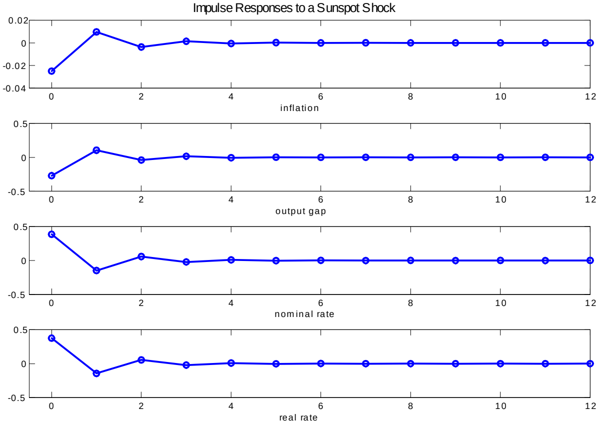
- A forward-looking rule would be:

$$i_t = r_t^n + \phi_\pi E_t \pi_{t+1} + \phi_y E_t \widetilde{y}_{t+1}. \tag{29}$$

- The conditions are then:

$$\begin{aligned} \kappa\left(\phi_\pi - 1\right) + \phi_y\left(1-\beta\right) &> 0, \\ \kappa\left(\phi_\pi - 1\right) + \phi_y\left(1+\beta\right) &< 2\sigma\left(1+\beta\right) \end{aligned}$$

- The reaction cannot be too large. Why?
- Let us consider the impulse responses for $\phi_\pi = 40$ and $\phi_y = 0$.



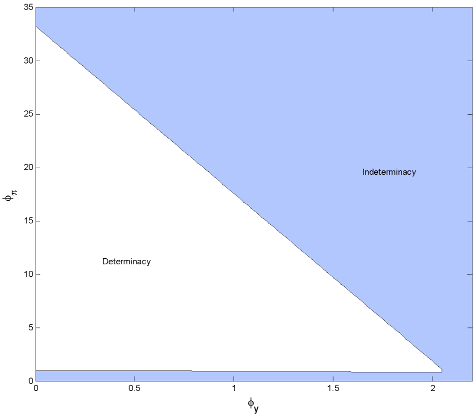
See Galí (1997), "Solving Linear Dynamic Models with Sunspot Equilibria: A note"

Optimal Monetary Policy in the Canonical Model

Indeterminacy – Forward-Looking Rule

- A too large response to expected inflation might give rise to self-fulfilling prophecies of oscillating dynamics:
  - Agents expect output to drop in the current period, but increase (above steady state) next period. Inflation follows the same pattern.
  - The central bank reacts strongly to the increase in expected inflation by increasing nominal interest rates such that the real interest rate drops much (so much that long real rates drop).
- Using our calibration (and letting $\phi_y = 0$) we see that $1 < \phi_\pi < 32.216$. This is well above the coefficients characterizing empirical interest rate rules.

Figure 4.2



Source: Galí (2008)

- While optimal interest rate rules appear to take a relatively simple form, there exists an important reason why they are unlikely to provide useful practical guidance for the conduct of monetary policy: require that the policy rate is adjusted one-for-one with the natural rate of interest.
- Assumes that the natural real interest rate can be observed, but that requires knowledge about:
    1. The economy's "true model".
    2. The values taken by all its parameters.
    3. The realized value (observed in real time) of all the shocks impinging on the economy.

- Have led many authors to propose a variety of "simple rules" – rules that a central bank could arguably adopt in practice – and to analyze their properties.
- The desirability of any given simple rule is thus given to a large extent by its robustness, i.e. its ability to yield a good performance across different models and parameter configurations.
- Svensson makes similar arguments for simple targeting rules (simple loss-functions).

- The average welfare loss per period is given by:

$$\mathbb{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) var\left(\widetilde{y}_t\right) + \frac{\epsilon}{\lambda} var\left(\pi_t\right)\right],$$

where we have taken the unconditional expectation of the (period) welfare function above.

- Evaluate two types of rules:
  - Taylor rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \phi_y \widehat{y}_t^n,$$

    where $\widehat{y}_t \equiv \log(Y_t / Y)$ is the log-deviation of output from its steady state.
  - Constant money growth rule:

$$\begin{aligned} \Delta m_t &= 0, \\ m_t - p_t &= y_t - \eta i_t + \zeta_t, \end{aligned}$$

    where ζ_t are money demand shocks and $\Delta\zeta_t = \rho_\zeta \Delta\zeta_{t-1} + \varepsilon_t^\zeta$.

Results: Galí (2008, Table 4.1)

Table 4.1: Evaluation of Simple Monetary Policy Rules

	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38