Two useful tricks for log-linearization

Gang Sun
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Trick 1: Log-linearizing the term $X_{1t}^{\alpha_1}X_{2t}^{\alpha_2}\cdots X_{nt}^{\alpha_n}$, we have

$$X_{1t}^{\alpha_1}X_{2t}^{\alpha_2}\cdots X_{nt}^{\alpha_n} \approx X_1^{\alpha_1}X_2^{\alpha_2}\cdots X_n^{\alpha_n}(1+\alpha_1 x_{1t}+\alpha_2 x_{2t}+\cdots \alpha_n x_{nt})$$

Example:
1. Production function

$$AK_t^{\alpha}L_t^{\beta} \approx AK^{\alpha}L^{\beta}(1+\alpha k_t+\beta l_t)$$

2. Real wage

$$W_t/P_t = W_t P_t^{-1} \approx W/P(1+w_t-p_t)$$

Trick 2: Log-linearizing the equation $X_{1t}^{\alpha_1}X_{1t}^{\alpha_2}\cdots X_{nt}^{\alpha_n}+\cdots+Y_{1t}^{\beta_1}Y_{2t}^{\beta_2}\cdots Y_{mt}^{\beta_m}+$

$A=0$, where A is constant, we have

$$(X_1^{\alpha_1}\cdots X_n^{\alpha_n})(\alpha_1 x_{1t}+\cdots+\alpha_n x_{nt})+\cdots+(Y_1^{\beta_1}\cdots Y_m^{\beta_m})(\beta_1 y_{1t}+\cdots+\beta_n y_{nt}) \approx 0$$

Proof.

$$\begin{aligned}
0 &= X_{1t}^{\alpha_1}X_{2t}^{\alpha_2}\cdots X_{nt}^{\alpha_n}+\cdots+Y_{1t}^{\beta_1}Y_{2t}^{\beta_2}\cdots Y_{mt}^{\beta_m}+A \\
&\approx \underbrace{X_1^{\alpha_1}\cdots X_n^{\alpha_n}+\cdots+Y_1^{\beta_1}\cdots Y_m^{\beta_m}+A}_{=0} \\
&\quad +X_1^{\alpha_1}\cdots X_n^{\alpha_n}(\alpha_1 x_{1t}+\cdots+\alpha_n x_{nt})+\cdots+Y_1^{\beta_1}\cdots Y_m^{\beta_m}(\beta_1 y_{1t}+\cdots+\beta_n y_{nt}) \\
&= X_1^{\alpha_1}\cdots X_n^{\alpha_n}(\alpha_1 x_{1t}+\cdots+\alpha_n x_{nt})+\cdots+Y_1^{\beta_1}\cdots Y_m^{\beta_m}(\beta_1 y_{1t}+\cdots+\beta_n y_{nt})
\end{aligned}$$

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Example
1. Labor supply

$$\begin{aligned}
\Omega_t &= vC_t(1-N_t)^{-1} \\
\Omega_t-\Omega_t N_t &= vC_t \\
\Omega\omega_t-\Omega N(\omega_t+n_t) &\approx vCc_t \\
&\cdots\cdots
\end{aligned}$$

Notice we cancel out the value on steday state.
2. Price index

$$\begin{aligned}
(\frac{P_t}{P_{t-1}\Pi})^{1-\varepsilon} &= \theta+(1-\theta)(\frac{P_t^*}{P_{t-1}\Pi})^{1-\varepsilon} \\
(1-\varepsilon)(p_t-p_{t-1}-\pi) &\approx (1-\theta)(1-\varepsilon)(p_t^*-p_{t-1}-\pi) \\
&\cdots\cdots
\end{aligned}$$

Notice the constant term θ dissappears.