Monetary Policy

(Advanced Monetary Economics)

ECON 4325

Nina Larsson Midthjell - Lecture 3 - 29 January 2016

Two versions of the Gali book

- Unfortunately, this semester we have to deal with two versions of the Gali book due to a misunderstanding at the book store.
- Some of you have the 2008 version, some have the 2015 version
- The 2015 version has some added material and also some changes in parameter notation. The slides will be informative about this as we go along.
- For the added material, all necessary information will be on the slides so that the 2008 book + the slides will be sufficient for those of you with the old book.

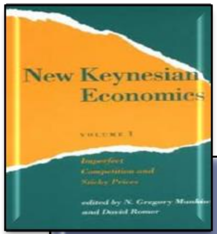
Outline

- Introduction to a classical monetary model
- Households
  - The representative household solves a dynamic optimization problem
- Firms
  - The representative firm's technology is introduced – determines the firm's optimal behavior under the assumption of price and wage-taking (remember: PC)
- Market Clearing
  - Equilibrium. Shows how real variables are uniquely determined independent of monetary policy
- Log-linearization and equilibrium dynamics
- Equilibrium behavior of nominal variables
  - Introducing monetary policy rules
- Money in the Utility Function

Introduction
to a classical monetary model

1980-90's: Flexible Price

- The RBC theory
  - Establishes the use of DSGE models as a central tool (micro foundation)
  - Perfect Competition and fully flexible prices and wages
  - Business cycles were seen as efficient
  - Technology shocks important (Supply side driven economy)
  - Limited role of monetary policy
  - Rational Expectations – The Lucas Critique
- The Classical Monetary Model
  - Introduces a monetary sector
  - Still fully flexible p and w
  - Neutrality of monetary policy wrt real variables
  - Not a very popular belief among central bankers (obviously)
  - A conflict between theoretical prediction and empirical evidence and normative implication and policy practice: Changes in monetary policy seem to influence output and employment in the short run



1990-2000's: New Keynesian Models

- Motivated by the shortcomings of the flexible price models
- Still intertemporal utility maximization in a DSGE framework (microfoundation)
- Introduces monopolistic competition in product and factor markets to make it more realistic (In contrast to social planner who seeks to clear all markets at all times)
- Introduces nominal rigidities, which leads to non-neutrality of monetary policy in the short run.
- Classical long run properties
  - Prices and wages adjust and economy goes back to natural equilibrium

Introduction to a classical monetary model

- Soon we will be able to understand:
  - What drives the real wage
  - What is the relationship between the real interest rate and output
- Households:
  - Complete financial markets
  - Perfectly competitive labor market
- Firms:
  - Competitive firms (Monopolistic competition and price rigidity are still not introduced)
  - Production function with labor as only input

$$Y_t = A_t N_t^{1-\alpha}$$

- General Equilibrium
  - DSGE-model

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Solutions added

Everything related to Z_t is added in the 2015 version

Households 1/11

- The representative household:
  - Lives forever
  - Chooses labor supply, consumption and one-period bonds
  - Maximizes discounted expected utility:

Correction: «+k» added to the Z

$$E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}; Z_{t+k}) \quad (1)$$

- C = Consumption　　N = Employment (1-leisure)
- β = The discount factor　　U = Period utility
- **Z** = An exogenous preference shifter (added in the 2015 version)
- $U_{cz,t} \equiv \frac{\delta^2 U_t}{\delta C_t \delta Z_t} > 0$ = An increase in Z_t raises the marginal utility of consumption

Solutions added

D_t (= dividends, accruing to households as firm owners) in the 2015 version replaces T_t (subsidies) from the 2008 version

Households 2/11

- The objective function is maximized subject to:

The sequence of budget constraints (Flow BC)

$$P_{t+k}C_{t+k} + Q_{t+k}B_{t+k} \leq B_{t+k-1} + W_{t+k}N_{t+k} + D_{t+k}, \qquad k \geq 0 \qquad (2)$$

- P = Price consumption good
- Q = Price on one-period risk free nominal bond
- B = One period risk free nominal bond that pays one nominal unit of money on maturity
- W = Nominal wage
- **D =** dividends, accruing to households in their condition of firm owners

= taken as given. Why?

How do we interpret this constraint?

Λ_t (= the stochastic discount factor) is added in equation 3 in the 2015 version

Households 3/11

- ...and subject to:

The solvency constraint: $$\lim_{T\to\infty} E_t\left\{\Lambda_{t,T}\frac{B_T}{P_T}\right\} \geq 0 \quad (3)$$

The present discounted value of wealth at infinity is non-negative: makes sure that the household pays back debt (cannot have positive debt in end period) . $\Lambda_{t,T} \equiv \beta^{T-t} U_{c,T} / U_{c,t}$ is the stochastic discount factor.

- The HH maximization problem:

Correction: «+k» added to the Z

Maximize: $$E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}; Z_{t+k}) \quad (1)$$

subject to: $$P_{t+k}C_{t+k} + Q_{t+k}B_{t+k} \leq B_{t+k-1} + W_{t+k}N_{t+k} + D_{t+k} \quad (2)$$

and: $$\lim_{T\to\infty} E_t\left\{\Lambda_{t,T}\frac{B_T}{P_T}\right\} \geq 0 \quad (3)$$

Z_t (preference shifter, see slide 7) and a specification of what happens if $\sigma = 1$ is added in the 2015 version

Households

- Next step: Specify the period utility function
  - We work with both **separable** and **non-separable** utility

Separable:

$$U(C_t, N_t; Z_t) = \begin{cases} \left(\dfrac{C_t^{1-\sigma} - 1}{1-\sigma} - \dfrac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \dfrac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

Non-separable:

$$U(C_t, N_t) = \frac{\left[C_t (1 - N_t)^{\nu} \right]^{1-\sigma} - 1}{1-\sigma}$$

In the following, we assume that $\sigma \neq 1$, $\sigma \geq 0$ and $\varphi \geq 0$

Households 5/11

- Separable period utility:

$$U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t \qquad (4)$$

- $\frac{1}{\sigma}$ is the intertemporal elasticity of substitution: measures how willing the household is to substitute consumption over time when the real interest rate changes
- $\frac{1}{\varphi}$ is the (Frisch) labor supply elasticity: measures how willing the household is to substitute leisure for hours worked when the real wage changes

- Let us now solve for HH optimal consumption and labor supply by maximizing (1) with respect to (2), under separable period utility as described in (4)

This slide is added to supplement the 2008 version: All on slide is additions in the 2015 version

Households 6/11

- The analysis is considerably simplified by two properties of the utility functions:
  - (i) Separability (i.e. the cross derivative is zero), and
  - (ii) The implied constancy of σ and φ , which leads to simple log-linearized approximations to the equilibrium conditions.

- The preference shifter $\boldsymbol{Z_t}$ should be interpreted as a **shock to the effective discount factor** (which becomes $Z_t\beta^t$, for t = 0, 1, 2,.....), whose effect will be restricted to **intertemporal** choices (through its effect on the intertemporal marginal rate of substitution), but with no effect on **intratemporal** choices (since it does not affect the intratemporal marginal rate of substitution). The two mentioned marginal rates of substitution are explained on slides 15-16

- We assume the shock $z_t \equiv \log Z_t$ to follow an exogenous AR(1) process (see seminar 1 for details about AR(1)s)

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z, \quad 0 < \rho_z < 1$$

Solutions added

Households 7/11

- The max problem is: $$\text{Max}E_t \sum_{k=0}^{\infty} \beta^k \left[\frac{C_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi} \right] Z_{t+k}$$

subject to: $$P_{t+k}C_{t+k} + Q_{t+k}B_{t+k} = B_{t+k-1} + W_{t+k}N_{t+k} + D_{t+k}$$

- The Lagrangian:

$$L_t = E_t \sum_{k=0}^{\infty} \beta^k \left[\left(\frac{C_{t+k}^{1-\sigma} - 1}{1-\sigma} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi} \right) Z_{t+k} - \lambda_{t+k} \left(P_{t+k}C_{t+k} + Q_{t+k}B_{t+k} - B_{t+k-1} - W_{t+k}N_{t+k} - D_{t+k} \right) \right]$$

where: $\hat{\lambda}_{t+k} = \beta^k \lambda_{t+k}$

Solutions added

Households 8/11

- The first order conditions:

$$\lambda_t = \frac{C_t^{-\sigma}}{P_t} Z_t = \frac{\partial U / \partial C_t}{P_t} \qquad (6)$$

$$\lambda_t = \frac{N_t^{\varphi}}{W_t} Z_t = \frac{-\partial U / \partial N_t}{W_t} \qquad (7)$$

$$Q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\} \qquad (8)$$

Solutions added

Households 9/11

- The optimality conditions:

The **INTRA**temporal optimality condition (combine (6) and (7)):

$$\frac{W_t}{P_t} = \Omega_t = \frac{N_t^{\varphi} Z_t}{C_t^{-\sigma} Z_t} = \frac{-\partial U / \partial N_t}{\partial U / \partial C_t} \qquad (9)$$

The **INTER**temporal optimality condition:

$$Q_t = \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \right\} = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\} = \beta E_t \left\{ \frac{\partial U / \partial C_{t+1}}{\partial U / \partial C_t} \Pi_{t+1}^{-1} \right\} \qquad (10)$$

Households

- Interpretation of the optimality conditions:

Households 11/11

Psssst!

- Smart to work on optimizing **non-separable** utility, as described on slide 10, as well

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Firms 1/2

- The representative firm has access to the following production technology:

$$Y_t = A_t N_t^{1-\alpha} \tag{11}$$

where Y_t and N_t are production and labor input, and $\log A_t \equiv a_t = \rho_a a_{t-1} + \varepsilon_t^a$. $0 < \rho_a < 1$

- The firm is <u>price-taker</u> in all markets

(i.e. the labor market and the goods market)

Firms 2/2

- The firm maximizes profits:

$$\max_{Y_t, N_t} \left[P_t Y_t - W_t N_t \right], \tag{12}$$

subject to:

$$Y_t = A_t N_t^{1-\alpha} \tag{13}$$

- FOC:

$$\frac{W_t}{P_t} = (1-\alpha) \frac{A_t}{N_t^{\alpha}} \tag{14}$$

Also equal to:

$$\boxed{MPL_t \equiv (1-\alpha) \frac{Y_t}{N_t} = \Omega_t.}$$

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Market Clearing 1/2

- All markets clear:

$$
\begin{aligned}
Y_t &= C_t\,, && (15)\\
N_t^s &= N_t^d = N_t\,. && (16)
\end{aligned}
$$

- Zero net savings: $B_t = 0$

This is all we say about market clearing at this point

Recall that a preference shock is added in the 2015 version

Market Clearing 2/2

- Will later discuss: *equilibrium dynamics*

Note:

1. Equilibrium dynamics of employment, output and the real interest rate are determined independently of monetary policy in this model: *Neutrality of monetary policy*

2. In this simple model: Technology and preferences only driving forces

3. In contrast to real variables, nominal variables (e.g. inflation and nominal interest rate) <u>cannot be determined uniquely by real forces</u> – requires a specification about how monetary policy is conducted

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Log-linearization

Why do we log-linearize?

- Easier to work with linear expressions
- Makes more sense economically to work with percentages
- More statistical reason: Way to stabilize the variance
- Useful method to approximate and solve dynamic models
- If taking logs yields a linear expression directly = precise description of relationship between variables: $y_t = a_t + \alpha k_t + (1 - \alpha)\, n_t.$

- If not = Approximation by log-linearization around steady state:

$$y_t = a_t + \alpha k_t + (1 - \alpha)\, n_t.$$

Log-linearization 2/12

Introducing the *three loglin steps* for log-linearization around steady state:

1. Manipulate the expression

2. First order Taylor Approximation around steady state

3. Solve for the variable of interest (and interpret the result)

Log-linearization 3/12

Log-linearizing the intratemporal optimality condition

- Labor supply: $$\Omega_t = C_t^{\sigma} N_t^{\varphi} \quad (9)$$

- Just taking logs yields a meaningful relationship: $$\omega_t = \sigma c_t + \varphi n_t \quad (17)$$

- Interpretation:

Log-linearization 4/12

Log-linearizing the intratemporal optimality condition around steady state:

- Labor supply: $$\Omega_t = C_t^{\sigma} N_t^{\varphi} \qquad (9)$$

Step 1 – Manipulate the expression

$$\Rightarrow \frac{\Omega\Omega_t}{\Omega} = \left(\frac{CC_t}{C}\right)^{\sigma}\left(\frac{NN_t}{N}\right)^{\varphi}$$

$$\Rightarrow \Omega e^{\omega_t} = C^{\sigma} e^{\sigma c_t} N^{\varphi} e^{\varphi n_t}$$

where

$$e^{\log\left(\frac{\Omega_t}{\Omega}\right)} = \frac{\Omega_t}{\Omega},$$

$$\log\left(\frac{\Omega_t}{\Omega}\right) = \omega_t \approx \frac{\Omega_t - \Omega}{\Omega}$$

$$\Rightarrow e^{\omega_t} = e^{\sigma c_t + \varphi n_t}$$

NB!! We always work with the natural log **ln**, not the common log, so when I, or Gali in his book(s), write "log" we mean ln, the natural logarithm

Solutions added

Log-linearization 5/12

Step 2 – Taylor approximation around steady state:

If X is the steady state, then: $$F(X_t) \approx F(X) + F'(X)(X_t - X)$$

For the manipulated labor supply schedule:

LHS: $$e^{\omega_t} \approx e^0 + e^0(\omega_t - 0) \Rightarrow e^{\omega_t} \approx 1 + \omega_t$$

RHS: $$e^{\sigma c_t + \varphi n_t} \approx e^0 + \sigma e^0(c_t - 0) + \varphi e^0(n_t - 0)$$
$$\Rightarrow e^{\sigma c_t + \varphi n_t} \approx 1 + \sigma c_t + \varphi n_t$$

Together: $$\omega_t = \sigma c_t + \varphi n_t$$

Solutions added

Log-linearization 6/12

Step 3 – Solving for variables of interest:

$$\omega_t = \sigma c_t + \varphi n_t \qquad (17)^*$$

Remember: $$\omega_t = \sigma c_t + \varphi n_t \qquad (17)$$

Why do we get the same expression?

No need for an approximation around steady state (17)* when taking logs directly (17) gives us a meaningful relationship.

Recall: Z_t has been added to the utility function in the 2015 version (see slide 11, equation 4)

Log-linearization 7/12

Log-linearizing the intertemporal optimality condition

Consumption Euler Equation: $$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \right) \left(\frac{Z_{t+1}}{Z_t} \right) \Pi_{t+1}^{-1} \right\} \tag{10}$$

Because $E_t(\log X_t) \neq \log(E_t X_t)$, a log-linear **approximation** is required.

$\boxed{\text{Always the natural log}}$

Step 1 – Manipulate the expression

$$\beta = \frac{1}{1+\rho} \Rightarrow \log \beta = \log(1) - \log(1+\rho) \approx -\rho,$$

$$Q_t = \frac{1}{1+i_t} \Rightarrow \log Q_t = \log(1) - \log(1+i_t) \approx -i_t$$

Recall: Z_t has been added to the utility function in the 2015 version (see slide 11, equation 4)

Log-linearization 8/12

Step 1 – Manipulate the expression continues

$$Q_t = \beta E_t\left\{\left(\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\right)\left(\frac{Z_{t+1}}{Z_t}\right)\Pi_{t+1}^{-1}\right\} \Rightarrow 1 = E_t\left\{\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{Z_{t+1}}{Z_t}\right)\Pi_{t+1}^{-1}Q_t^{-1}\right\}$$

$$\Rightarrow e^0 = E_t\left\{e^{[-\rho-\sigma c_{t+1}+\sigma c_t+z_{t+1}-z_t-\pi_{t+1}+i_t]}\right\} \Rightarrow e^0 = E_t\left\{e^{[-\rho-\sigma\Delta c_{t+1}+\Delta z_{t+1}-\pi_{t+1}+i_t]}\right\}$$

Why not just move the Expectation term inside the brackets?

Defining steady state values as in Gali: γ, π and i yields:

$$e^0 = e^{[-\rho-\sigma\gamma-\pi+i]} \Rightarrow i-\pi = \sigma\gamma+\rho$$

Recall: Z_t has been added to the utility function in the 2015 version (see slide 11, equation 4)

Log-linearization 9/12

Step 2 – Taylor approximation around steady state:

RHS:

$$E_t\left\{e^{[-\rho-\sigma\Delta c_{t+1}+\Delta z_{t+1}-\pi_{t+1}+i_t]}\right\}\approx e^0-e^0E_t(\rho-\rho)-\sigma e^0E_t(\Delta c_{t+1}-\gamma)$$

$$+e^0E_t(\Delta z_{t+1}-0)-e^0E_t(\pi_{t+1}-\pi)+e^0E_t(i_t-i)$$

$$\approx e^0-\sigma E_t\Delta c_{t+1}+\sigma\gamma-(1-\rho_z)z_t-E_t\pi_{t+1}+\pi+i_t-i$$

$$\approx 1-\sigma E_t\Delta c_{t+1}-(1-\rho_z)z_t-E_t\pi_{t+1}+i_t-(i-\pi-\sigma\gamma)$$

$$\approx 1-\sigma E_t\Delta c_{t+1}-(1-\rho_z)z_t-E_t\pi_{t+1}+i_t-\rho$$

Recall: Z_t has been added to the utility function in the 2015 version (see slide 11, equation 4)

Log-linearization 10/12

Solutions added

Step 2 – Taylor approximation around steady state con't:

LHS and RHS together:

$$1 = 1 - \sigma E_t \Delta c_{t+1} - (1 - \rho_z) z_t - E_t \pi_{t+1} + i_t - \rho$$

$$\Rightarrow 0 = -\sigma E_t \Delta c_{t+1} - (1 - \rho_z) z_t - E_t \pi_{t+1} + i_t - \rho$$

Recall: Z_t has been added to the utility function in the 2015 version (see slide 11, equation 4)

Log-linearization 11/12

Step 3 – Solving and interpreting

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} - \rho\right] + \frac{1}{\sigma}(1-\rho_z)z_t \qquad (18)$$

Log-linearization 12/12

- Log-linearizing the firm's production function and optimality condition:

$$\boxed{y_t = a_t + (1-\alpha)\, n_t,} \tag{19}$$

$$w_t - p_t = \omega_t = y_t - n_t + \log(1-\alpha) \tag{20}$$

where (20) can be interpreted as the labor demand schedule, mapping the real wage into the quantity of labor demanded, given the level of technology.

Remember the FOC Firms:

$$\boxed{MPL_t \equiv (1-\alpha)\,\frac{Y_t}{N_t} = \Omega_t.}$$

Why are eq (19) and (20) not linearized
around steady state? Will the equilibrium change if they were?

Equilibrium dynamics 1/10

Solving the model for real variables

- Remember that all output is consumed in equilibrium:

$$\boxed{Y_t = C_t ,} \qquad y_t = c_t \qquad (21)$$

- In order to solve the model for real variables we need the log-linear household and firm optimality conditions, in addition to the log-linear aggregate production relationship.

- 5 endogenous variables and 5 equations yield a unique solution for the equilibrium dynamics of the following real variables:

Output, consumption, employment, real interest rate, real wage

Recall: Z_t has been added to the utility function in the 2015 version (see slide 11, equation 4)

Equilibrium dynamics 2/10

Solving the model for real variables

- The five equations are:

(17) $$\omega_t = \sigma c_t + \varphi n_t$$ Labor supply condition

(18) $$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} - \rho\right] + \frac{1}{\sigma}(1-\rho_z) z_t$$ Cons.Euler

(19) $$\boxed{y_t = a_t + (1-\alpha)\, n_t,}$$ Firm prod. function

(20) $$\omega_t = y_t - n_t + \log(1-\alpha)$$ Labor demand cond.

(21) $$y_t = c_t$$ Market clearing

Equilibrium dynamics 3/10

Solving the model for real variables

- 5 steps to easily (it's true!) solve the model for real variables:

Step 1

- Combine the log-linear labor supply and labor demand optimality conditions (i.e. equation (17) and (20) and solve for employment

$$n_t$$

Step 2

- Insert for n_t in the log-linear aggregate production function (equation (19)) and find equilibrium output

$$y_t^* \qquad = c_t^*$$

Step 3

- Solve for equlibrium real interest rate r_t^* by solving the log-linear Euler equation (equation (18)) for r_t knowing that:

$$r_t \equiv i_t - E_t\{\pi_{t+1}\}$$

- Next, insert for Δy_{t+1}^* found from: $y_{t+1}^* - y_t^*$

Equilibrium dynamics 4/10

Solving the model for real variables

- After 3 steps we have found equilibrium output and equilibrium real interest rate.

Step 4

- Insert for equilibrium output y_t^* into the solution for n_t found under step 1. This yields equlibrium employment n_t^*

Step 5

- Solve either the labor supply optimality condition (equation (17)) or the labor demand condition (equation (20)) for the real wage by inserting for equilibrium output and employment. This yields equilibrium real wage. ω_t^*

Equilibrium dynamics 5/10

Solving the model for real variables

- Give it a GO!

Equilibrium dynamics 6/10

Solving the model for real variables

- Give it a GO!

Equilibrium dynamics 7/10

Solving the model for real variables

- Give it a GO!

Equilibrium dynamics 8/10

Solving the model for real variables

- Solutions and interpretation (no space to write? See next slide):

$$c_t^* = y_t^* \qquad (22)$$

$$y_t^* = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} = \psi_{ya} a_t + \xi_y \qquad (23)$$

$$r_t^* = \rho + (1-\rho_z) z_t + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} \qquad (24)$$

$$n_t^* = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} = \psi_{na} a_t + \xi_n \qquad (25)$$

$$\omega_t^* = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{[\sigma(1-\alpha)+\varphi]\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} = \psi_{\omega a} a_t + \xi_\omega \qquad (26)$$

Equilibrium dynamics 9/10

Solving the model for real variables

- Interpretation of solutions

Solutions added

Equilibrium dynamics 10/10

Solving the model for real variables

"Classical Dichotomy"

- The equilibrium dynamics of employment, output and the real interest rate is determined independently of monetary policy in this model:

Neutrality of monetary policy

- In this simple model: Technology only driving force of all real variables
- The preference shock only affects the equilibrium real interest rate

In contrast to real variables:

- Nominal variables (e.g. inflation and nominal interest rate) <u>cannot be determined uniquely by real forces</u>

- Requires a specification about how monetary policy is conducted

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For the exogenous path for the nominal interest rate and the simple interest rate rule: Study the version in your respective book. The exogenous path for the money supply has no changes.

Equilibrium behavior of nominal variables 1/5

Introducing monetary policy rules

- The Fisherian equation for the nominal interest rate:

$$i_t = E_t\{\pi_{t+1}\} + r_t \qquad (27)$$

The nominal interest rate adjusts one-for-one with expected inflation, given a real interest rate determined exclusively by real factors (independently of monetary policy rules (equation 24)

- Introducing monetary policy rules in order to determine equilibrium price behavior
  - **Exogenous path for the nominal interest rate** $i_t = E_t\{\pi_{t+1}\} + r_t$
    - Leads to price level indeterminacy (and money supply and nominal wage indeterminacy)
  - **A simple Inflation-Based interest rate rule** $i_t = \rho + \phi_t \pi_t$
    - Determinate equilibrium if Taylor principle satisfied
  - **An exogenous path for the money supply**
    - Determines the price level uniquely

Equilibrium behavior of nominal variables 2/5

Introducing monetary policy rules – An exogenous path for Money Supply

- Introducing function for real money demand:

$$m_t - p_t = y_t - \eta i_t \tag{28}$$

Combining equation (28) and the Fisherian equation (27) eliminates the nominal interest rate and yields the following expression for the price level (remember that $\pi_{t+1} = p_{t+1} - p_t$)

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t \tag{29}$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$ evolves independently of the money supply path $\{m_t\}$

Equilibrium behavior of nominal variables 3/5

Introducing monetary policy rules – An exogenous path for Money Supply

- Solving (29) forward, assuming $\eta > 0$, yields:

$$p_t = \left(\frac{1}{1+\eta}\right)\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k E_t\{m_{t+k}\} + u_t^{'}$$

- We want to rewrite the expression in terms of expected future growth rate of nominal money:

$$p_t = \underbrace{m_t - m_t}_{add\,\&\,subtract} + \frac{1}{1+\eta}m_t + \underbrace{\frac{\eta}{1+\eta}E_t m_{t+1} - \frac{\eta}{1+\eta}E_t m_{t+1}}_{add\,\&\,subtract} + \frac{1}{1+\eta}\frac{\eta}{1+\eta}E_t m_{t+1}$$

$$\underbrace{+\left(\frac{\eta}{1+\eta}\right)^2 E_t m_{t+2} - \left(\frac{\eta}{1+\eta}\right)^2 E_t m_{t+2}}_{add\,\&\,subtract} + \frac{1}{1+\eta}\left(\frac{\eta}{1+\eta}\right)^2 E_t m_{t+2} + \ldots +$$

Equilibrium behavior of nominal variables 4/5

Introducing monetary policy rules – An exogenous path for Money Supply

- We can then write:

$$p_t = m_t + \frac{\eta}{1+\eta} E_t \Delta m_{t+1} + \frac{\eta}{1+\eta} \left(\underbrace{\frac{1}{1+\eta} - 1}_{\frac{-\eta}{1+\eta}} \right) E_t m_{t+1} + \left(\frac{\eta}{1+\eta} \right)^2 E_t m_{t+2} + \left(\frac{\eta}{1+\eta} \right)^2 \left(\frac{1}{1+\eta} - 1 \right) E_t m_{t+2} + \ldots +$$

- And then write: $p_t = m_t + \frac{\eta}{1+\eta} E_t \Delta m_{t+1} + \left(\frac{\eta}{1+\eta} \right)^2 E_t m_{t+2} - \left(\frac{\eta}{1+\eta} \right)^2 E_t m_{t+1} + \ldots +$

Summing up yields:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + u_t^{'} \qquad (30)$$

An arbitrary exogenous path for the money supply will always determine the price level uniquely. Assuming an AR(1) for money growth:

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

And assuming absence of real shocks (as in book), equation (30) can be solved backward so that we get the following expression for the price level in equilibrium:

Equilibrium behavior of nominal variables 5/5

Introducing monetary policy rules – An exogenous path for Money Supply

$$p_t = m_t + \frac{\eta \rho_m}{1+\eta(1-\rho_m)} \Delta m_t \tag{31}$$

> If $\rho_m > 0$ (the parameter is often calibrated to 0.5 based on empirical evidence), the price level should respond more than one-for-one with the increase in the money supply.
>
> This prediction is in stark contrast to the sluggish response of the price level observed in empirical estimates of the effects of monetary policy shocks.

- What is optimal monetary policy?
  - Hard to say, no rule seems to be more desirable than another- HH only cares about C and hours
  - Can be overcome by introducing money in the utility function
  - The model cannot explain the observed effect of monetary policy on real variables ! → **Introduce nominal rigidities next time (NKM)**

Outline

- Introduction to a classical monetary model
- Households
  - The representative household solves a dynamic optimization problem
- Firms
  - The representative firm's technology is introduced – determines the firm's optimal behavior under the assumption of price and wage-taking (remember: PC)
- Market Clearing
  - Equilibrium. Shows how real variables are uniquely determined independent of monetary policy
- Log-linearization and equilibrium dynamics
- Equilibrium behavior of nominal variables
  - Introducing monetary policy rules
- Money in the Utility Function

Money in the Utility Function

DROPPED

ergo

Section 2.5 in Gali is taken out of the course

Next week and the week after

- Next week: No lecture. Use the time to catch up
- The week after (on February 12): Galí chapter 3 (Not section 3.4.2)
  - The New Keynesian Model

Monetary Policy
(Advanced Monetary Economics)

ECON 4325