Monetary Policy

(Advanced Monetary Economics)

ECON 4325

Outline

- Short summary of lecture 3, with interpretation of equilibrium dynamics
- Introduction to a basic New Keynesian model – What is new?
  - Introducing price rigidities
- Deriving the model – households
- Deriving the model – firms
- Start on market clearing

About chapter 3:

- For lecture 4: Galí chapter 3, pages 41-46 (2008) / pages 52-59 (2015)
- For lecture 6: Galí chapter 3, pages 46-56 (2008) / pages 59-74 (2015)
- Both books: Section 3.4.2 may be skipped.

We start by interpreting the equilibrium dynamics in the classical model derived at lecture 3 so:

MAKE SURE TO BRING YOUR LECTURE 3 SLIDES!

Λ_t (= the stochastic discount factor) and Z_t (=preference shifter) are added in the 2015 version, see lecture 3 for more details

Short summary of lecture 3 1/7

Households

- Households maximization problem. Maximize: $E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}; Z_{t+k})$

  subject to $P_{t+k}C_{t+k} + Q_{t+k}B_{t+k} \leq B_{t+k-1} + W_{t+k}N_{t+k} + D_{t+k}$ and $\lim_{T\to\infty} E_t\left\{\Lambda_{t,T}\frac{B_T}{P_T}\right\} \geq 0$

- We specified a separable utility function (for $\sigma \neq 1$): $U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right)Z_t$

- Solving this problem gave us the following optimality conditions for HH behavior:
  - The intratemporal optimality condition: $\frac{W_t}{P_t} = \Omega_t = \frac{N_t^{\varphi}Z_t}{C_t^{-\sigma}Z_t} = \frac{N_t^{\varphi}}{C_t^{-\sigma}} = \frac{-\partial U/\partial N_t}{\partial U/\partial C_t}$
  - The intertemporal (Euler) optimality condition:

$$Q_t = \beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\right\} = \beta E_t\left\{\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{Z_{t+1}}{Z_t}\frac{P_t}{P_{t+1}}\right\} = \beta E_t\left\{\frac{\partial U/\partial C_{t+1}}{\partial U/\partial C_t}\Pi_{t+1}^{-1}\right\}$$

Short summary of lecture 3 — 2/7

Firms and market clearing

- The firm maximizes profits: $\max_{Y_t, N_t} \; [P_t Y_t - W_t N_t]$, subject to: $Y_t = A_t N_t^{1-\alpha}$
- Resulting in the optimality condition: $MPL_t \equiv (1-\alpha)\dfrac{Y_t}{N_t} = \Omega_t .$
- All markets clear:

$$\begin{aligned} Y_t &= C_t, \\ N_t^s &= N_t^d = N_t . \end{aligned}$$

- Then, we log-linearized the household and firm optimality conditions, and the aggregate production relationship, in order to solve for equilibrium dynamics of real variables (around steady state).
- 5 endogenous variables and 5 equations gave us a unique solution for the equilibrium dynamics of output, consumption, employment, the real interest rate and the real wage.

Short summary of lecture 3 3/7

Equilibrium dynamics of real variables

The equations were:

$$\omega_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}\left[i_t - E_t \pi_{t+1} - \rho\right] + \frac{1}{\sigma}(1-\rho_z) z_t$$

$$y_t = a_t + (1-\alpha)\, n_t,$$

$$\omega_t = y_t - n_t + \log(1-\alpha)$$

$$y_t = c_t$$

The solution told us that:

- Output and the real wage always increase with a positive shock to technology
- Ambiguous effect on employment
- The development of the real interest rate depends critically on output growth and hence on the evolution of technology

Short summary of lecture 3 4/7

Interpretation of equilibrium dynamics

- Solutions and interpretation (no space to write? See next slide):

Eq numbers from last week

$$c_t^* = y_t^* \qquad (22)$$

$$y_t^* = \frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi} a_t + \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\alpha+\varphi} = \psi_{ya} a_t + \xi_y \qquad (23)$$

$$r_t^* = \rho + (1-\rho_z) z_t + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} \qquad (24)$$

$$n_t^* = \frac{1-\sigma}{\sigma(1-\alpha)+\alpha+\varphi} a_t + \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\alpha+\varphi} = \psi_{na} a_t + \xi_n \qquad (25)$$

$$\omega_t^* = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\alpha+\varphi} a_t + \frac{[\varphi+\sigma(1-\alpha)]\log(1-\alpha)}{\sigma(1-\alpha)+\alpha+\varphi} = \psi_{\omega a} a_t + \xi_\omega \qquad (26)$$

Short summary of lecture 3 5/7
Interpretation of equilibrium dynamics

Short summary of lecture 3 6/7

Interpretation of equilibrium dynamics

Short summary of lecture 3 7/7

Results

Found "Classical Dichotomy"

- Equilibrium dynamics of employment, output and the real interest rate is determined independently of monetary policy in this model – *Neutrality of monetary policy*

- Technology only driving force of all real variables

- Introduced monetary policy specification in order to say something about equilibrium dynamics of nominal variables

- Did not result in an optimal monetary policy - no rule seemed to be more desirable than another

- The model could not explain the observed effect of monetary policy on real variables

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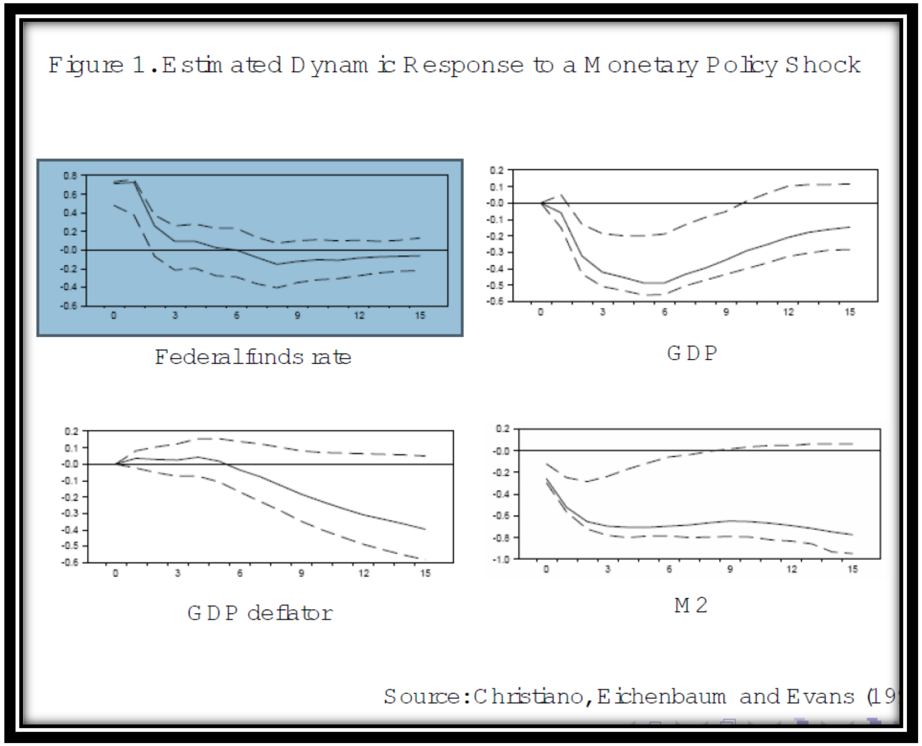
Introduction to a basic New Keynesian model 1/3

- Failure of the classical model - Empirical evidence on:
  - persistent effects on real variables of monetary policy shocks
  - slow adjustment of the aggregate price level

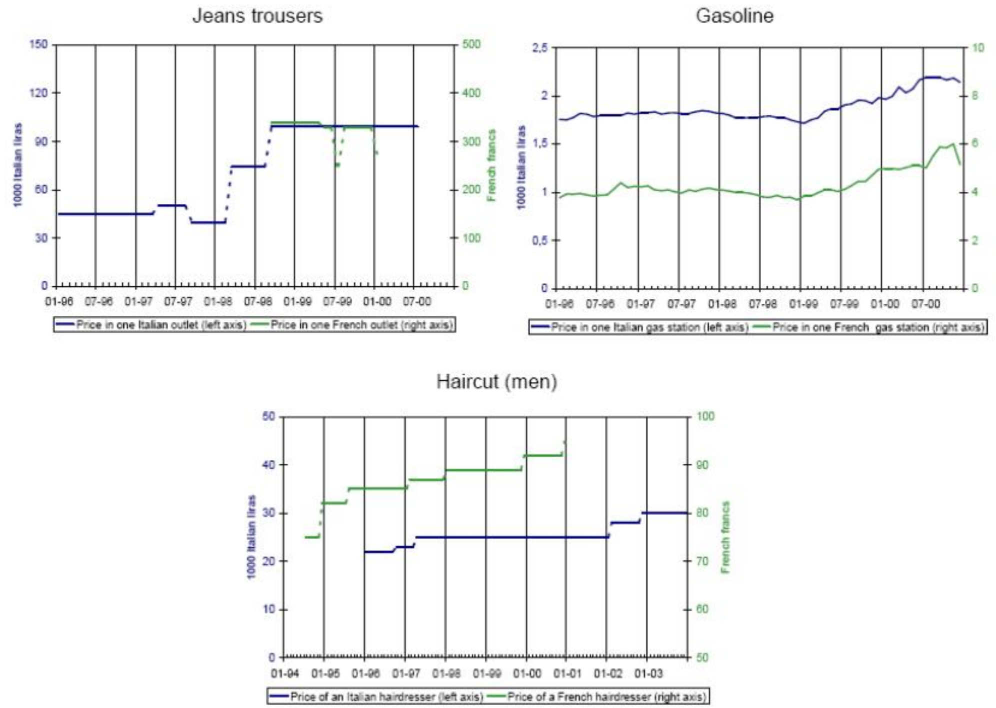
  - Tightening of monetary policy
  - Hump-shaped decline in GDP
  - Flat response of the GDP-deflator for over 1 year, then it declines
    - Price rigidity
  - Liquidity effect



Introduction to a basic New Keynesian model 2/3

- A large amount of heterogeneity in price durations across sectors/types of goods
- Largest degree of rigidity: Services
- Smallest degree of rigidity: Energy, unprocessed food.
- Median price stickiness: 8-11 months
- If interested: Check out staff memo about this at NB by Solveig Erlandsen

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. W P

Introduction to a basic New Keynesian model 3/3

- **What is old news:**
  - DSGE modeling with:
    - Profit maximizing firms
    - Utility maximizing households
  - Complete financial markets
  - Perfectly competitive labor markets (for now)

- **What is news:**
  - Monopolistically competitive firms
    - Firms set their own price
    - Prices are sticky
  - Implication:
    - Monetary Policy has real consequences in the short run!

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Deriving the model 1/11
Households

- Maximize consumption and leisure given an intertemporal budget constraint.
    - New: A continuum of differentiated goods!
- Optimality conditions:
    - Intratemporal
        - Allocation between consumption and leisure (as before).
        - Allocation between different types of goods.
- Intertemporal:
    - The consumer Euler condition (as before).

Info about monetary holdings mentioned on this slide was added in the 2015 version

Deriving the model 2/11

Households

- Maximize discounted expected utility: $E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}, N_{t+k}; Z_{t+k})$

where: $$C_t \equiv \left(\int_0^1 (C_t(i))^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \qquad (1)$$

- Period budget constraint: $$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \qquad (2)$$

- Solvency constraint like before (lecture 3)

- **New info in the 2015 version:** "*Note that monetary holdings are not modeled explicitly, so one can think of the present framework as the cashless limit of an economy with money in the utility function, with the latter being additively separable.*"

Deriving the model 3/11

Households

- Household decision making:
  - Must decide how much labor to supply

    As before
  - Must decide how to smooth consumption over time

    As before
  - Must decide how to allocate its consumption expenditures among different goods!

    NEW!
  - The household must maximize the consumption index C_t for any given level of expenditures:

$$\int_0^1 P_t(i)\, C_t(i)\, di \, .$$

The given expenditure level is denoted Z_t in the 2008 version of the book. Changed to X_t in the 2015 version, to avoid confusion with the preference shifter Z_t (see slide 4).

Deriving the model 4/11

Households

- Assuming a given expenditure level: $$\int_0^1 P_t(i) C_t(i) di \equiv X_t \qquad (3)$$

- The HH maximization problem is: $$\underset{C_t}{Max} \underbrace{\left\{ \int_0^1 (C_t(i))^{1-\frac{1}{\varepsilon}} di \right\}^{\frac{\varepsilon}{\varepsilon-1}}}_{C_t}$$

subject to: $$\int_0^1 P_t(i) C_t(i) di = X_t$$

- The Lagrangian: $$L_t = \left\{ \int_0^1 (C_t(i))^{1-\frac{1}{\varepsilon}} di \right\}^{\frac{\varepsilon}{\varepsilon-1}} - \lambda_t \left\{ \int_0^1 P_t(i) C_t(i) di - X_t \right\}$$

Deriving the model 5/11
Households

- First order condition:

$$\frac{dL_t}{dC_t(i)} = \frac{\varepsilon}{\varepsilon-1}\left\{\int_0^1 (C_t(i))^{1-\frac{1}{\varepsilon}} di\right\}^{\frac{\varepsilon}{\varepsilon-1}-1}(1-\frac{1}{\varepsilon})(C_t(i))^{-\frac{1}{\varepsilon}} - \lambda_t P_t(i) = 0, \qquad \forall i \in [0,1]$$

$$\Rightarrow \left\{\int_0^1 (C_t(i))^{1-\frac{1}{\varepsilon}} di\right\}^{\frac{1}{\varepsilon-1}}(C_t(i))^{-\frac{1}{\varepsilon}} - \lambda_t P_t(i) = 0, \qquad \forall i \in [0,1]$$

$$\Rightarrow \underbrace{\left\{\int_0^1 (C_t(i))^{1-\frac{1}{\varepsilon}} di\right\}^{\frac{1}{\varepsilon-1}}}_{C_t^{1/\varepsilon}}(C_t(i))^{-\frac{1}{\varepsilon}} - \lambda_t P_t(i) = C_t^{1/\varepsilon}(C_t(i))^{-\frac{1}{\varepsilon}} - \lambda_t P_t(i) = 0, \forall i \in [0,1]$$

$$\Rightarrow C_t(i) = C_t\left[\lambda_t P_t(i)\right]^{-\varepsilon}, \qquad \forall i \in [0,1]$$

Deriving the model 6/11
Households

- Then for any good j: $C_t(j) = C_t\left[\lambda_t P_t(j)\right]^{-\varepsilon}$

Solving for λ_t :

$$[\lambda_t]^{\varepsilon} = \frac{C_t}{C_t(j)P_t(j)^{\varepsilon}} \Rightarrow \lambda_t = \left[\frac{C_t}{C_t(j)}\right]^{\frac{1}{\varepsilon}} P_t(j)^{-1}$$

inserting for λ_t in the first order condition for good i yields:

(4) $$C_t(i) = C_t(j)\left[\frac{P_t(i)}{P_t(j)}\right]^{-\varepsilon}, \quad \text{for} \quad \forall i, j \in [0,1] \quad \text{and} \quad i \neq j$$

- Interpretation:

Deriving the model 7/11
Households

What do we have so far?

- HH optimal behavior regarding allocation between goods at given prices and for given level of expenditure X_t.

- Inserting for optimal consumption of good *i* (eq. 4) in the expression for consumption expenditure (eq. 3) yields:

$$\int_0^1 P_t(i) \underbrace{C_t(j)\left[\frac{P_t(i)}{P_t(j)}\right]^{-\varepsilon}}_{C_t(i)} di = X_t \Rightarrow \frac{C_t(j)}{P_t(j)^{-\varepsilon}} = \frac{X_t}{\int_0^1 P_t(i)^{1-\varepsilon}\, di}, \qquad \forall i \in [0,1],\ i \neq j$$

- Inserting for $\frac{C_t(j)}{P_t(j)^{-\varepsilon}}$ back into the first order condition for good *i* (eq. (4)) yields:

$$C_t(i) = \frac{X_t}{\int_0^1 P_t(i)^{1-\varepsilon}\, di} P_t(i)^{-\varepsilon} = \frac{X_t}{P_t}\left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon}, \qquad \forall i \in [0,1] \qquad (5)$$

When assuming that the aggregate price index P_t equals: $\left[\int_0^1 P_t(i)^{1-\varepsilon}\, di\right]^{\frac{1}{1-\varepsilon}}$

Deriving the model 8/11

Households

What do we have now?

- HH optimal choice of consumption of good *i* for given prices and expenditure (whatever that is).

- Inserting for optimal consumption of good *i* (eq. 5) in the consumption index (eq. 1):

$$C_t = \left\{ \int_0^1 \underbrace{\left(\frac{X_t}{P_t} \left[\frac{P_t(i)}{P_t} \right]^{-\varepsilon} \right)}_{optimal C_t(i)}^{1-\frac{1}{\varepsilon}} di \right\}^{\frac{\varepsilon}{\varepsilon-1}} = \frac{X_t}{P_t^{1-\varepsilon}} \left\{ \int_0^1 \left(\left[P_t(i) \right]^{1-\varepsilon} \right) di \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \forall i \in [0,1]$$

$$\Rightarrow P_t C_t = X_t P_t^{\varepsilon} \left(P_t^{1-\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}} = X_t P_t^{\varepsilon} \left(\frac{1}{P_t^{\varepsilon-1}} \right)^{\frac{\varepsilon}{\varepsilon-1}} = X_t P_t^{\varepsilon} P_t^{-\varepsilon} = X_t,$$

- Inserting for the expenditure level from equation (3) yields:

$$P_t C_t = \int_0^1 P_t(i) C_t(i) di, \quad \forall i \in [0,1] \qquad (6)$$

Deriving the model 9/11
Households

What does equation (6) indicate?

- When HHs behave optimally, then total consumption expenditures (RHS) can be written as the product of the price index and consumption index (LHS)

- Knowing that $P_t C_t = X_t$ in optimum, we can insert for $\frac{X_t}{P_t}$ in equation (5) to get the following set of demand equations:

(7) $$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t, \qquad \forall i \in [0,1]$$

- Knowing that $P_t C_t = \int_0^1 P_t(i) C_t(i) di,$ is true when HH behaves optimally, we can do the following:

Deriving the model 10/11
Households

- We can rewrite the budget constraint as:

$$(8) \qquad P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

- Let us use the separable utility function:

$$(9) \qquad U(C_t, N_t; Z_t) = \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t$$

- The remaining first-order conditions associated with the household problem are:

$$(10) \qquad \frac{W_t}{P_t} = \Omega_t = \frac{N_t^{\varphi}}{C_t^{-\sigma}}$$

$$(11) \qquad Q_t = \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{Z_{t+1}}{Z_t} \frac{P_t}{P_{t+1}} \right\}$$

- We also have: $i_t \equiv -\log Q_t$

Deriving the model 11/11
Households

- The log-linear versions of the household optimality conditions:

(12) $$\omega_t = \sigma c_t + \varphi n_t$$

(13) $$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}\left(1 - \rho_z\right) z_t$$

together with the set of optimal demand equations:

(7) $$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t ,$$

are the equations we bring with us from the household-part of the model in order to solve for equilibrium behavior of real and nominal variables

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Deriving the model 1/13

Firms

- There is a continuum of monopolistically competitive firms, indexed by $i \in [0,1]$.
- Each firm produces a differentiated good.
- Identical production technology:

$$Y_t(i) = A_t N_t(i)^{1-\alpha}, \qquad (14)$$

where $Y_t(i)$ and $N_t(i)$ are firm i's production and labor input, and $\ln A_t \equiv a_t = \rho_a a_{t-1} + \varepsilon_t^a$.

- Let us denote the marginal product of labor for firm i as $MPN_t(i) \equiv (1-\alpha) Y_t(i) / N_t(i)$.

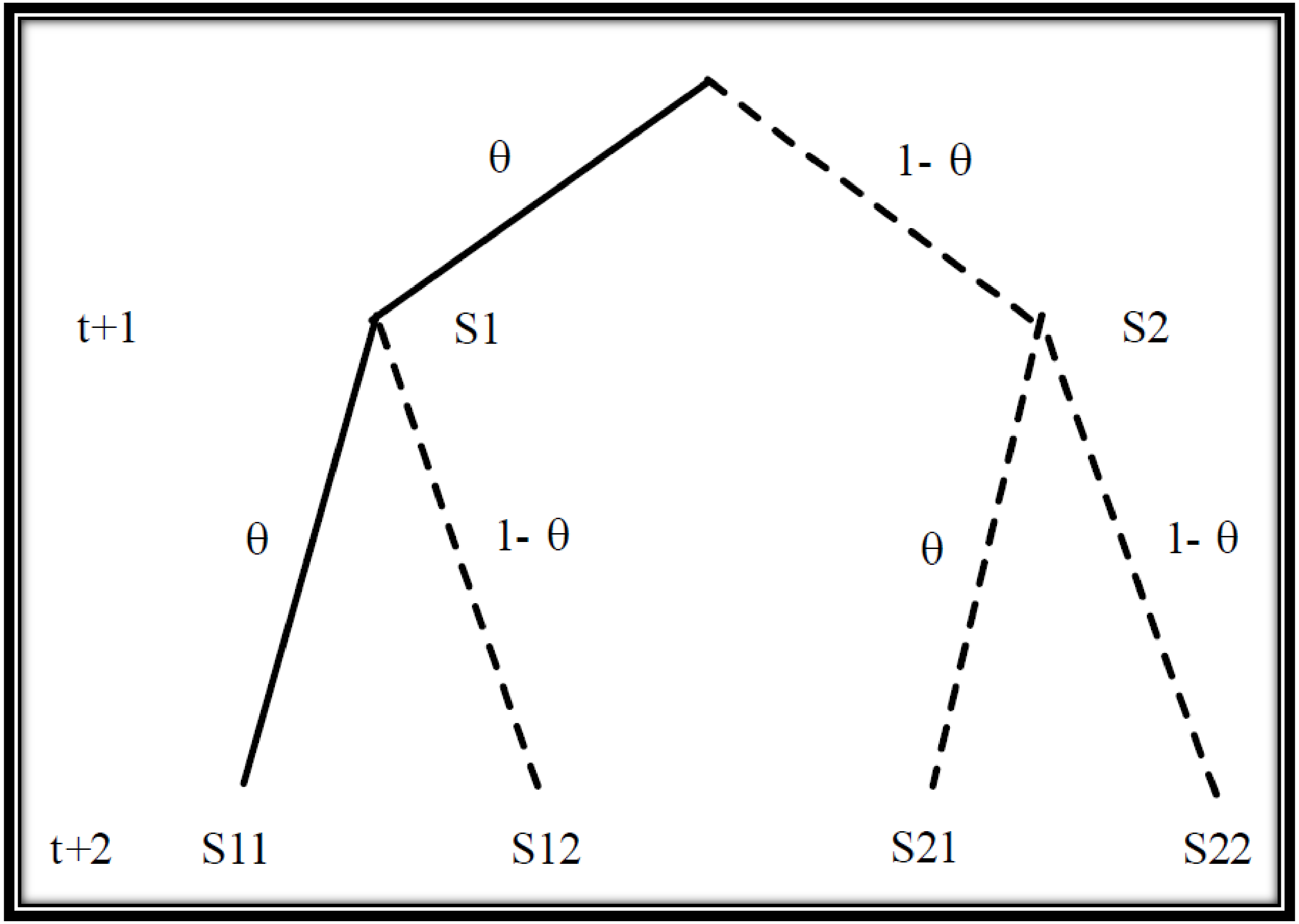
Deriving the model 2/13

Firms

- We assume staggered price setting à la Calvo (1983): each firm faces a constant and exogenous probability, $(1-\theta)$, of getting to reoptimize its price in any given period.
- Fraction $(1-\theta)$ of firms change their price in any given period.
- On average firms change their price every $\frac{1}{1-\theta}$ period.

Deriving the model 3/13

Firms



Deriving the model 4/13

Firms

What drives aggregate inflation when there is Calvo pricing?

- Aggregate price index: $P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$
- Firms not resetting prices in period *t*: $S(t) \subset [0,1]$
- Firms resetting choose optimal price P_t^*, then:

$$P_t = \left[\int_{S(t)} P_{t-1}(i)^{1-\varepsilon} di + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} = \left[\theta (P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

- Solving for gross inflation $\Pi_t = \frac{P_t}{P_{t-1}}$ yields: $\Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left[\frac{P_t^*}{P_{t-1}} \right]^{1-\varepsilon}$

Log-linear appr. around ss: $\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$ (15)

Or equivalently: $p_t = \theta p_{t-1} + (1-\theta) p_t^*$

Steady State gross inflation =1

Change in notation for the nominal cost function from Ψ_{t+k} (2008) to ς_{t+k}(2015). Also, change in the max-problem from $Q_{t,t+k}$ (2008) to $^{\Lambda_{t,t+k}}/_{P_{t+k}}$ (2015). The latter notation makes more sense, see next slide.

Deriving the model 5/13

Firms

- Firms choose prices, output and labor input to maximize:

$$\underset{P_t^*}{Max} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \left(\frac{1}{P_{t+k}} \right) \left(P_t^* Y_{t+k|t} - \varsigma_{t+k} \left(Y_{t+k|t} \right) \right) \right\}$$

subject to:

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}, \qquad (16)$$

$$Y_{t+k|t} = A_{t+k} N_{t+k|t}^{1-\alpha} \qquad (14)$$

For each firm *i*:

$$P_{t+k+1}(i) = \begin{cases} P_{t+k+1}^*(i) & \text{with probability} \quad (1-\theta) \\ P_{t+k}(i) & \text{with probability} \quad \theta \end{cases}$$

$\Lambda_{t,t+k} \equiv \beta^k \frac{U_{c,t+k}}{U_{c,t}}$ (the stochastic discount factor), ς_{t+k} =(nominal) cost function

The change in the firm max-problem mentioned on the previous slide is an improvement. We will use this new version in the course. The additions in the 2015 version of the book related to this change is some additional explanatory text and an appendix. Both are given on this slide.

Deriving the model 6/13

Firms

The explanatory text (on page 56):

$$\underset{P_t^*}{Max}\sum_{k=0}^{\infty}\theta^k E_t\left\{\Lambda_{t,t+k}\left(\frac{1}{P_{t+k}}\right)\left(P_t^* Y_{t+k|t}-\varsigma_{t+k}\left(Y_{t+k|t}\right)\right)\right\}$$

for $k = 0, 1, 2, \dots$ where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is the (nominal) cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t. Note that it is implicitly assumed that the firm always meets the demand for its good at the current price. That assumption, which is maintained throughout the analysis below, requires, in turn, that the average price markup is sufficiently large and/or that the shifts in demand resulting from a variety of shocks are not too large.

The appendix (on page 84):

3.3 Firm's Objective Function

The value of a firm in period t, expressed in terms of current consumption is given by

$$V_t(i) = \sum_{k=0}^{\infty} E_t\{\Lambda_{t,t+k}(D_{t+k}(i)/P_{t+k})\}$$

where $D_t(i) \equiv P_t(i)Y_t(i) - \mathcal{C}_t(Y_t(i))$.

Note that for a firm resetting its price in period t,

$$E_t\{\Lambda_{t,t+k}(D_{t+k}(i)/P_{t+k})\} = \theta^k E_t\{\Lambda_{t,t+k}(D_{t+k|t}/P_{t+k})\} + (1-\theta)\sum_{h=1}^{k}\theta^{k-h}E_t\{\Lambda_{t,t+k}(D_{t+k|t+h}/P_{t+k})\} \tag{37}$$

where $D_{t+k|t} \equiv P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t})$ denotes period $t+k$ dividends conditional on the price having been last reset in period t. Note that the second term on the right-hand side of (37) is independent of P_t^* since it involves states of nature for which the price has been reset at least once after period t.

Thus, the value of a firm resetting its price in period t is given by

$$V_{t|t} = \sum_{k=0}^{\infty}\theta^k E_t\{\Lambda_{t,t+k}(D_{t+k|t}/P_{t+k})\} + \Upsilon_t$$

where Υ_t is a term independent of P_t^* and hence can be ignored when choosing the latter, as reflected in the objective function in the main text.

Change in notation for the nominal unit cost from ψ_{t+k} (2008) to Ψ_{t+k} (2015).

Deriving the model 7/13

Firms

- The Lagrangian for firm *i*:

$$L_t = E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left(\frac{1}{P_{t+k}} \right) \Bigg[P_t^*(i) Y_{t+k|t}(i) - W_{t+k} N_{t+k|t}(i) - \zeta_{t+k|t}(i) \left(Y_{t+k|t}(i) - \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right) - \Psi_{t+k|t}(i) \left(Y_{t+k|t}(i) - A_{t+k} N_{t+k|t}(i)^{1-\alpha} \right) \Bigg]$$

$\Psi_{t+k|t}(i)$ is the shadow price on the production constraint, it gives the firm's nominal **unit cost** when prices were last set in period *t*: $\Psi_{t+k|t}(i) \equiv \varsigma'_{t+k}\left(Y_{t+k|t}(i)\right)$

$\zeta_{t+k|t}(i)$ is the shadow price on the demand constraint, it tells us the firm's nominal **unit profit** when prices were last set in period *t*

Deriving the model 8/13

Firms

First order condition with respect to hours:

$$\frac{\partial L_t}{\partial N_t(i)} = -W_t + \Psi_t(i)MPN_t(i) = 0 \Rightarrow \Psi_t(i) = \frac{W_t}{MPN_t(i)} \qquad (17)$$

First order condition with respect to output:

$$\frac{\partial L_t}{\partial Y_t(i)} = P_t^*(i) - \zeta_t(i) - \Psi_t(i) = 0 \Rightarrow \zeta_t(i) = P_t^*(i) - \Psi_t(i) \qquad (18)$$

First order condition with respect to the price (for firms re-optimizing in period t):

$$\frac{\partial L_t}{\partial P_t^*(i)} = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[Y_{t+k|t}(i) - \zeta_{t+k|t}(i) \left(\varepsilon \left[\frac{P_t^*(i)}{P_{t+k}} \right]^{-\varepsilon-1} \frac{Y_{t+k}}{P_{t+k}} \right) \right] \right\} = 0$$

Knowing that:

$$\zeta_{t+k|t}(i) \left(\varepsilon \left[\frac{P_t^*(i)}{P_{t+k}} \right]^{-\varepsilon-1} \frac{Y_{t+k}}{P_{t+k}} \right) = \varepsilon \zeta_{t+k|t}(i) \underbrace{\left(\left[\frac{P_t^*(i)}{P_{t+k}} \right]^{-\varepsilon} Y_{t+k} \right)}_{Y_{t+k|t}(i)} \left(\left[\frac{P_t^*(i)}{P_{t+k}} \right]^{-1} \frac{1}{P_{t+k}} \right) = \frac{\varepsilon \zeta_{t+k|t}(i) Y_{t+k|t}(i)}{P_t^*(i)}$$

yields:

Deriving the model 9/13

Firms

$$\frac{\partial L_t}{\partial P_t^*(i)} = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} \left[Y_{t+k|t}(i) - \frac{\varepsilon \zeta_{t+k|t}(i) Y_{t+k|t}(i)}{P_t^*(i)} \right] \right\} = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[1 - \frac{\varepsilon \zeta_{t+k|t}(i)}{P_t^*(i)} \right] \right\} = 0$$

$$\Rightarrow \frac{\partial L_t}{\partial P_t^*(i)} = \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[P_t^*(i) - \varepsilon \zeta_{t+k|t}(i) \right] \right\} = 0$$

Inserting for $\zeta_{t+k|t}(i)$ from the FOC for output:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[P_t^*(i) - \varepsilon \left(P_t^*(i) - \Psi_{t+k|t}(i) \right) \right] \right\} = 0$$

$$\Rightarrow \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[P_t^*(i)(1-\varepsilon) + \varepsilon \Psi_{t+k|t}(i) \right] \right\} = 0 \Rightarrow \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[P_t^*(i) - \underbrace{\frac{\varepsilon}{(\varepsilon-1)}}_{\mathrm{M}} \Psi_{t+k|t}(i) \right] \right\} = 0$$

$$\Rightarrow \sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[P_t^*(i) - \mathrm{M} \Psi_{t+k|t}(i) \right] \right\} = 0 \qquad (19)$$

Deriving the model 10/13
Firms

Firm's optimal behavior for choosing a price in period t hence satisfies:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t}(i) \left[P_t^*(i) - \mathrm{M}\Psi_{t+k|t}(i) \right] \right\} = 0 \qquad (19)$$

- If prices were fully flexible, then $(\theta = 0)$, every firm reset prices every period and the FOC reduces to:

$$P_t^*(i) = \mathrm{M}\Psi_t(i)$$

- With sticky nominal prices: The price is set as a mark-up over a weighted average of current and future expected marginal costs.
  - The future gets a lower weight, both due to discounting and the probability of a new price.
  - Periods with high demand get a high weight.

The log-linearization of equation (20) is presented a little differently in the two versions of the book, however, the resulting main equation is the same, only with some difference in notation. We will use a merge of the two versions, as presented on slides 38 and 39, because that provides the most information.

- We now know how firms will set *prices* in every period, but we want to know more about what drives *inflation* in a model with price rigidities. Hence, we rewrite equation (19):

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\frac{\Lambda_{t,t+k}}{P_{t+k}}Y_{t+k|t}(i)\left[\frac{P_t^*(i)}{P_{t-1}}-\frac{P_{t+k}}{P_{t+k}P_{t-1}}\mathrm{M}\Psi_{t+k|t}(i)\right]\right\}=$$

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\frac{\Lambda_{t,t+k}}{P_{t+k}}Y_{t+k|t}(i)\left[\frac{P_t^*(i)}{P_{t-1}}-\mathrm{M}MC_{t+k|t}(i)\Pi_{t-1,t+k}\right]\right\}=0 \qquad (20)$$

where $\Pi_{t-1,t+k}=\dfrac{P_{t+k}}{P_{t-1}}$ and $MC_{t+k|t}(i)=\dfrac{\Psi_{t+k|t}(i)}{P_{t+k}}$ (real marginal cost for a firm *i* which last set prices in period *t*)

- Assuming zero inflation in steady state, log-linearizing eq. 20 around steady state (NB: a hand-out on how to go from eq.(20) to eq.(21) is given at the lecture) yields:

$$p_t^*-p_{t-1}=(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{m}c_{t+k}+(p_{t+k}-p_{t-1})\} \qquad (21)$$

- The subscript (i) is dropped, as $P_t^*(i)$ will be the same for all firms that get to change prices in period *t.*

Some additional interpretation of equation (22) is provided in the (2015)-version of the book, see quote on the slide for full interpretation.

Deriving the model 12/13

Firms

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{m}c_{t+k} + (p_{t+k} - p_{t-1}\} \qquad (21)$$

- Equation (21) tells us what drives inflation and is our point of departure in order to construct the New Keynesian Phillips curve (which we will do at lecture 6).

- Rearranging terms, equation (21) becomes:

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{t+k}\} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\} \qquad (22)$$

where $\psi_{t+k|t} = \log(\Psi_{t+k|t})$, i.e. the log of the nominal unit cost.

- *"To the extent that prices are sticky ($\theta > 0$), firms set prices in a forward-looking way. The chosen price corresponds to their desired mark-up over a weighted average of their current and expected (nominal) marginal costs, proportional to the probability of the price remaining at each horizon, θ^k, times the cumulative discount factor, β^k"*

Deriving the model 13/13

Firms

- In order to solve for equilibrium we bring with us the following two equations from the firm's problem:

- The production function:

$$Y_t\left(i\right) = A_t N_t\left(i\right)^{1-\alpha}, \tag{14}$$

- The log-linear optimal price-setting condition:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\hat{m}c_{t+k} + (p_{t+k} - p_{t-1}\right\} \tag{21}$$

Outline

- Short summary of lecture 3, with interpretation of equilibrium dynamics
- Introduction to a basic New Keynesian model – What is new?
  - Introducing price rigidities
- Deriving the model – households
- Deriving the model – firms
- Start on market clearing

Deriving the model 1/2

Market Clearing

- Market clearing implies:

$$Y_t = C_t \tag{22}$$

$$\begin{aligned} N_t &= \int_0^1 N_t(i)\, di = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}} di \\ &= \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\alpha}} di \end{aligned} \tag{23}$$

Note the following:

- Firms set prices and production is demand determined (Keynesian assumption). For each type of goods $Y_t(i) = C_t(i)$.
- Demand for labor is given by the production function.

- In this simple model: consumption is the only source of demand for goods

Deriving the model 2/2

Market Clearing

- Taking logs of equation (23) yields: $(1-\alpha)n_t = y_t - a_t + d_t$

  where d_t measures price dispersion across firms.

- d_t will be equal to zero up to a first order approximation in the neighborhood of steady state, hence we assume the following approximate relation between aggregate output, employment and technology:

Rearranging terms: $y_t = a_t + (1-\alpha)n_t$

which can be thought of as determining aggregate employment, given aggregate output and technology.

$$n_t = \frac{1}{(1-\alpha)}(y_t - a_t) \qquad (24)$$

NB: Finding d_t is not required.

Next three weeks

- **Next week:** Guest lecture by chief economist Kjetil Olsen
- **On 26 February**: Winter break
- **On 4 March**: Equilibrium in the NKM
  - Finding the **NK Phillips curve**
  - Finding the **Dynamic IS equation**
  - Closing the model by **introducing monetary policy** – We'll see that monetary policy *has* effect on real variables in the short run
  - Learning mathematical method nr. 2 (nr. 1 was log-linearization around steady state): ***Method of undetermined coefficients***
- About chapter 3:
  - For lecture 4: Galí chapter 3, pages 41-46 (2008) / pages 52-59 (2015)
  - For lecture 6: Galí chapter 3, pages 46-56 (2008) / pages 59-74 (2015)
  - Both books: Section 3.4.2 may be skipped.
- Seminar 2: The exercise is on the web

Monetary Policy
(Advanced Monetary Economics)

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