Monetary Policy
(Advanced Monetary Economics)

ECON 4325

Nina Larsson Midthjell - Lecture 5 - 19 February 2016

Outline

- Finishing up from lecture 4:
  - Slides 32-43

- Equilibrium in the New Keynesian Model
  - Equilibrium equations to work with
  - Equilibrium behavior of inflation
  - Introducing the output gap
  - Finding the New Keynesian Phillips Curve
  - Finding the Dynamic IS equation

- A New Keynesian treat☺

Finishing up from lecture 4

Remember your lecture 4 slides!

- Deriving the model for the firm (slides 32-40)
- Market clearing (slides 41-42)

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Equilibrium in the NKM
Equilibrium equations to work with

From the **household optimization problem**:

- The log-linear versions of the household optimality conditions:

(1) $$\omega_t = \sigma c_t + \varphi n_t$$ (Equation 12, lecture 4)

(2) $$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}\left(1 - \rho_z\right) z_t$$ (Equation 13, lecture 4)

together with the set of optimal demand equations:

(3) $$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t,$$ (Equation 7, lecture 4)

are the equations we bring with us from the household-part of the model in order to solve for equilibrium behavior of real and nominal variables

Equilibrium in the NKM

Equilibrium equations to work with

2/6

From the **firm optimization problem**:

- The production function:

(4) $$Y_t(i) = A_t N_t(i)^{1-\alpha},$$ (Equation 14, lecture 4)

together with the log-linear optimal price-setting condition

(5) $$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\hat{m}c_{t+k|t} + (p_{t+k} - p_{t-1}\right\}$$ (Equation 21, lecture 4)

are the equations we bring with us from the firm-part of the model in order to solve for equilibrium behavior of real and nominal variables

Equilibrium in the NKM

Equilibrium equations to work with

Lecture 4 also introduces the following market clearing conditions:

- Market clearing in the **goods market**:

(6) $$Y_t = C_t \Rightarrow y_t = c_t$$ (Equation 22, lecture 4)

- Market clearing in the **labor market** resulted in the following expression determining aggregate employment, given aggregate output and technology, when zero price dispersion across firms in the neighborhood of steady state is assumed:

(7) $$n_t = \frac{1}{(1-\alpha)}\left(y_t - a_t\right)$$ (Equation 24, lecture 4)

- Recall also that the aggregate inflation behavior around steady state is:

(8) $$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$$ (Equation 15, lecture 4)

Equilibrium in the NKM

Equilibrium equations to work with

- We are hence working with the following eight equations in order to describe and discuss **equilibrium behavior in the NKM**:

(1) $$\omega_t = \sigma c_t + \varphi n_t$$

(2) $$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

(3) $$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t,$$

(4) $$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

(5) $$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\}$$

(6) $$Y_t = C_t \Rightarrow y_t = c_t$$

(7) $$n_t = \frac{1}{(1-\alpha)}(y_t - a_t)$$

(8) $$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$$

Equilibrium in the NKM

Equilibrium equations to work with

- How to interpret the optimal price-setting condition (equation 5)?

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{m}c_{t+k|t} + (p_{t+k} - p_{t-1})\}$$

$$\Rightarrow p_t^* = p_{t-1} - \left[(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k p_{t-1}\right] - (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k mc + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{t+k}\}$$

$$\Rightarrow p_t^* = p_{t-1} - \left[\frac{(1-\beta\theta)}{(1-\beta\theta)}p_{t-1}\right] - \left[\frac{(1-\beta\theta)}{(1-\beta\theta)}mc\right] + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{t+k}\}$$

$$\Rightarrow p_t^* = \underbrace{-mc}_{\mu} + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{mc_{t+k|t} + p_{t+k}\} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\} \quad (5)$$

where $mc_{t+k|t} + p_{t+k} = \log(MC_{t+k|t}P_{t+k}) = \log\left(\frac{\Psi_{t+k|t}P_{t+k}}{P_{t+k}}\right) = \log(\Psi_{t+k|t}) = \psi_{t+k|t}$

, i.e. the log of the nominal unit cost, and μ is the log of the desired (frictionless) markup.

Equilibrium in the NKM

Equilibrium equations to work with 6/6

- Why is μ the log of the desired mark-up?
  - Equation 20 from lecture 4 (dropping subscript (i)) showed us how inflation relates to firms' optimal price setting behavior:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \frac{\Lambda_{t,t+k}}{P_{t+k}} Y_{t+k|t} \left[\frac{P_t^*}{P_{t-1}} - \mathrm{M} MC_{t+k|t} \Pi_{t-1,t+k} \right] \right\} = 0$$

In steady state the expression reduces to: $\sum_{k=0}^{\infty} \theta^k \left\{ \beta^k \frac{Y}{P} [1 - \mathrm{M} MC] \right\} = 0$

which indicates that:

$$\frac{1}{MC} = \mathrm{M} \Rightarrow \log(\mathrm{M}) = \log(1) - \log(MC) \Rightarrow \mu = -mc$$

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New info added in the 2015-version of the book is included on this slide

Typo correction on this slide. This expression is the correct one

Equilibrium in the NKM

Equilibrium behavior of inflation

We'll now work with the equilibrium equations 1-8 in order to say something about the behavior of inflation.

- First (for later purpose), combine equation (6) with equation (2):

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}\left(1-\rho_z\right)z_t \qquad (9)$$

- From the 2015-version: Note that equation (9) can be solved forward (try this yourself) to yield:

$$y_t = \frac{1}{\sigma}z_t - \frac{1}{\sigma}\sum_{k=0}^{\infty} E_t\{i_{t+k} - E_t\{\pi_{t+1+k}\} - \rho\} + \lim_{T\to\infty} E_t\{y_{t+T}\}$$

- From the 2015-version:

> Thus, an exogenous shock will impact output only to the extent that it meets one or more of the following conditions: (i) it shifts the preference parameter z_t, (ii) it has a permanent effect on the level of output, or (iii) it leads to a deviation of the real interest rate from the discount rate, current or anticipated.

The 2008 version focus on real mc, whereas the 2015 version focus on nominal marginal costs, hence I Include both definitions here so you see the difference, but we will continue with equation (10).

Equilibrium in the NKM

Equilibrium behavior of inflation

- Next, the individual firm's marginal product of labor (MPN) can be found from equation (4):

$$MPN_t(i) = (1-\alpha)\frac{A_t}{N_t(i)^{\alpha}} \Rightarrow mpn_t(i) = \log(1-\alpha) + a_t - \alpha n_t(i)$$

- All firms face the same technology and wages, hence average nominal marginal costs is given by:

$$\Psi_t = \frac{W_t}{MPN_t} \Rightarrow \psi_t = w_t - mpn_t = w_t - \log(1-\alpha) - (a_t - \alpha n_t) \qquad (10)$$

- Average real marginal costs is given by:

$$MC_t = \frac{W_t}{P_t MPN_t} \Rightarrow mc_t = w_t - p_t - mpn_t = w_t - p_t - \log(1-\alpha) - (a_t - \alpha n_t)$$

Equilibrium in the NKM

Equilibrium behavior of inflation

- Inserting for $n_t = \frac{y_t}{1-\alpha} - \frac{a_t}{1-\alpha}$ from equation (7) in equation (10) yields:

$$\psi_t = w_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \qquad (11)$$

- Hence, for a firm that last sat prices in period t, marginal costs equal:

$$\psi_{t+k|t} = w_{t+k} - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha)$$

$$\Rightarrow \psi_{t+k|t} = w_{t+k} - \log(1-\alpha) - \frac{1}{1-\alpha} a_{t+k} + \frac{\alpha}{1-\alpha} y_{t+k} - \frac{\alpha}{1-\alpha} y_{t+k} + \frac{\alpha}{1-\alpha} y_{t+k|t}$$

$$\Rightarrow \psi_{t+k|t} = \psi_{t+k} + \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right) \qquad (12)$$

- Interpretation:

Equilibrium in the NKM

Equilibrium behavior of inflation 4/10

- We would like to develop equation (12) a bit further (why?). Taking logs of equation (3) and inserting for the market clearing equation (6) yields:

$$c_t(i) = -\varepsilon[p_t(i) - p_t] + c_t = -\varepsilon[p_t(i) - p_t] + y_t \Rightarrow y_t = c_t(i) + \varepsilon[p_t(i) - p_t] \qquad (13)$$

- In period *t+k*, this equals:

$$y_{t+k} = c_{t+k}(i) + \varepsilon[p_{t+k}(i) - p_{t+k}],$$
$$y_{t+k|t} = c_{t+k}(i) + \varepsilon[p_{t+k}(i) - p_t^*],$$
$$\Rightarrow y_{t+k|t} - y_{t+k} = -\varepsilon[p_t^* - p_{t+k}]$$

- Inserting for this in equation (12) yields:

$$\psi_{t+k|t} = \psi_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}[p_t^* - p_{t+k}] \qquad (14)$$

Equilibrium in the NKM

Equilibrium behavior of inflation

- Recall equation (5): $p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\psi_{t+k|t}\}$

  and that: $\psi_{t+k} = mc_{t+k} + p_{t+k}$

- Equation (14) gave us an expression for a firm's nominal marginal costs when it last set prices in period *t*. Inserting for $\psi_{t+k|t}$ in eq. (5) yields:

$$p_t^* = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right)\right\} \Rightarrow p_t^* = \mu - (1-\beta\theta)\frac{\alpha\varepsilon}{1-\alpha}\sum_{k=0}^{\infty}(\beta\theta)^k p_t^* + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\psi_{t+k} + \left[\frac{\alpha\varepsilon}{1-\alpha}\right]p_{t+k}\right\}$$

$$\Rightarrow p_t^*\left[1+\frac{\alpha\varepsilon}{1-\alpha}\right] = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k} + \left[\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}\right]p_{t+k}\right\} \Rightarrow p_t^* = \left[\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right]\left[\mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{mc_{t+k} + \left[\frac{1-\alpha+\alpha\varepsilon}{1-\alpha}\right]p_{t+k}\right\}\right]$$

$$\Rightarrow p_t^* = \Theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left\{\left[\frac{1}{\Theta}\right]p_{t+k}\right\} - \Theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{\mu}_{t+k}\}, \qquad \Theta \equiv \left[\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right] \leq 1$$

where: $\mu + E_t\{mc_{t+k}\} = \mu - E_t\{\mu_{t+k}\} = -(E_t\{\mu_{t+k}\} - \mu) = -E_t\{\hat{\mu}_{t+k}\}$ because: $\mu_t = p_t - \psi_t = p_t - mc_t - p_t = mc_t$

$$\Rightarrow p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{p_{t+k}\} - \Theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{\mu}_{t+k}\},$$

- Subtracting p_{t-1} from both sides of the expression to discuss equilibrium behavior of inflation:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{p_{t+k} - p_{t-1}\} - \Theta(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{\mu}_{t+k}\}$$

- Rewriting the first term (using a little trick to get inflation terms):

$$(1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{p_{t+k} - p_{t-1}\} = (1-\beta\theta)(\underbrace{p_t - p_{t-1}}_{\pi_t}) + (1-\beta\theta)\beta\theta(\underbrace{E_t\{p_{t+1} - p_{t-1}\}}_{E_t\{\pi_{t+1}\}+\pi_t})$$

$$+ (1-\beta\theta)(\beta\theta)^2(\underbrace{E_t\{p_{t+2} - p_{t-1}\}}_{E_t\{\pi_{t+2}\}+E_t\{\pi_{t+1}\}+\pi_t}) + \ldots +$$

Collecting term on the RHS yields:

$$(1-\beta\theta)\left[\pi_t + \beta\theta E_t\{\pi_{t+1}\} + (\beta\theta)^2 E_t\{\pi_{t+2}\} + .. +\right] + (1-\beta\theta)\beta\theta\left[\pi_t + \beta\theta E_t\{\pi_{t+1}\} + (\beta\theta)^2 E_t\{\pi_{t+2}\} + .. +\right]$$

$$+ (1-\beta\theta)(\beta\theta)^2\left[\pi_t + \beta\theta E_t\{\pi_{t+1}\} + (\beta\theta)^2 E_t\{\pi_{t+2}\} + \ldots +\right] + \ldots +$$

$$\Rightarrow RHS = (1-\beta\theta)\left[\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\pi_{t+k}\}\right]\left(1 + \beta\theta + (\beta\theta)^2 + \ldots +\right) = \sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\pi_{t+k}\}$$

Equilibrium in the NKM

Equilibrium behavior of inflation

- Hence: $$p_t^* - p_{t-1} = \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\pi_{t+k}\} - \Theta(1-\beta\theta)\sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\hat{\mu}_{t+k}\} \quad (15)$$

- One period ahead: $$E_t\{p_{t+1}^* - p_t\} = \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\pi_{t+1+k}\} - \Theta(1-\beta\theta)\sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\hat{\mu}_{t+1+k}\}$$

- Inserting in equation (15) yields the recursive equation:

$$p_t^* - p_{t-1} = \pi_t + \beta\theta E_t\{p_{t+1}^* - p_t\} - \Theta(1-\beta\theta)\hat{\mu}_t \quad (16)$$

- We're getting close to explaining the equilibrium behavior of inflation!

- Recall the aggregate price dynamics from eq. (8).

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \Rightarrow p_t^* - p_{t-1} = \frac{1}{1-\theta}\pi_t$$

Equilibrium in the NKM

Equilibrium behavior of inflation 8/10

- We combine inflation behavior based on firms re-optimizing prices (i.e. eq.(16)) with the aggregate price dynamics from eq. (8).:

$$\frac{1}{1-\theta}\pi_t = \pi_t + \beta\theta E_t\left\{\frac{1}{1-\theta}\pi_{t+1}\right\} - \Theta(1-\beta\theta)\hat{\mu}_t \tag{17}$$

- Solving for π_t yields:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \underbrace{\frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta}_{\lambda}\hat{\mu}_t \tag{18}$$

- Interpretation (of all parts of the expression):

Equilibrium in the NKM
Equilibrium behavior of inflation 9/10

- Solving equation (18) forward yields:

$$\pi_t = \beta E_t\{\beta E_t\{\pi_{t+2}\} - \lambda E_t\{\hat{\mu}_{t+1}\}\} - \lambda\hat{\mu}_t = \beta^2 E_t\{\pi_{t+2}\} - \lambda\hat{\mu}_t - \beta\lambda E_t\{\hat{\mu}_{t+1}\}$$

$$\Rightarrow \pi_t = \underbrace{\beta^T E_t\{\pi_{t+T}\}}_{\lim_{T\to\infty}=0} - \lambda\left[\sum_{k=0}^{\infty}\beta^k E_t\{\hat{\mu}_{t+k}\}\right] = -\lambda\left[\sum_{k=0}^{\infty}\beta^k E_t\{\hat{\mu}_{t+k}\}\right] \qquad (19)$$

- Interpretation (How does it differ from equation (18)?):

Equilibrium in the NKM
Equilibrium behavior of inflation 10/10

- The Classical Monetary Model
  - Inflation is a result of movements in the aggregate price level, created by a monetary policy rule, in order to support an equilibrium allocation independent of the evolution of nominal variables
  - Does not take into account the mechanisms necessary to create the price level changes
- The New Keynesian model
  - Inflation is a result of aggregate consequences of carefully reasoned price-setting decisions made by firms based on their current and future cost conditions

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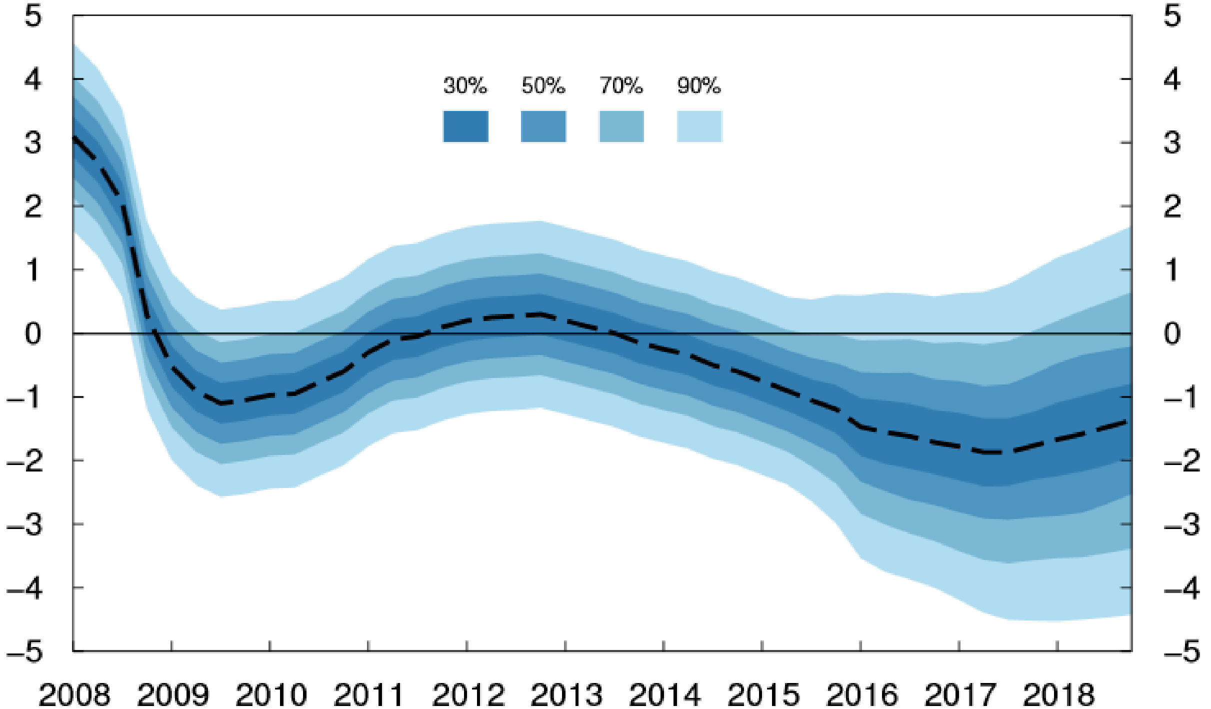
Equilibrium in the NKM
Introducing the output gap 1/5

Chart 2.4b Projected output gap[1)] in the baseline scenario with fan chart. Percent. 2008 Q1 – 2018 Q4



1) The output gap measures the percentage deviation between mainland GDP and projected potential mainland GDP.
Source: Norges Bank

Source: MPR 4/15

The main difference between the two book versions in this part of the chapter is that the 2008 version focuses on real marginal costs (mc), whereas the 2015 version focuses on the price mark-up, which equals (- mc). We follow the definitions in the 2015 version on the slides.

- Let's consider the relation between μ_t and aggregate economic activity, in order to find the output gap. Recall that $\mu_t = p_t - \psi_t$. Combined with equation (11), we have:

$$\mu_t = p_t - \psi_t = p_t - \left(w_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \right) = -\omega_t + \frac{1}{1-\alpha}(a_t - \alpha y_t) + \log(1-\alpha) \qquad (20)$$

- Now, let's make use of equation (1) (HH optimality condition) and insert in eq. (20) for the equilibrium real wage, and the market clearing condition (eq. 6):

$$\mu_t = -(\sigma y_t + \varphi n_t) + \frac{1}{1-\alpha}(a_t - \alpha y_t) + \log(1-\alpha)$$

- Again inserting for $n_t = \frac{y_t}{1-\alpha} - \frac{a_t}{1-\alpha}$ from equation (7) yields:

$$\mu_t = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) y_t + \frac{1+\varphi}{1-\alpha} a_t + \log(1-\alpha) \qquad (21)$$

Equilibrium in the NKM

Introducing the output gap

- Under flexible prices, the average mark-up is equal to the desired mark-up (i.e. the frictionless one). In other words, marginal costs are constant: $\mu = -mc$

- In every period there will be a natural level of output corresponding to that mark-up, for a given level of technology: $y_t^{natural}$

- Solving equation (21) for flexible prices yields an expression for the natural level of output:

$$\mu = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t^{natural} + \frac{1+\varphi}{1-\alpha} a_t + \log(1-\alpha) \qquad (22)$$

$$\Rightarrow y_t^{natural} = \underbrace{\frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}}_{\Psi_{ya}^{natural}} a_t - \underbrace{\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha}}_{\xi_y^{natural}} = \psi_{ya}^{natural} a_t + \xi_y^{natural} \qquad (23)$$

Equilibrium in the NKM

Introducing the output gap

- Recall equilibrium behavior of output in the classical model (See lecture note 3, slide 44, equation (23)):

$$y_t^* = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} = \psi_{ya} a_t + \xi_y$$

- The solution for the natural level of output in the NKM:

$$y_t^{natural} = \underbrace{\frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}}_{\Psi_{ya}^{natural}} a_t - \underbrace{\frac{(1-\alpha)(\mu-\log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha}}_{\xi_y^{natural}} = \psi_{ya}^{natural} a_t + \xi_y^{natural} \qquad (23)$$

- What are the similarities and differences??
  - **NB:** Wrong sign for $\xi_y^{natural}$ in both books. The correct sign is *negative.* Why?
  - What is the main difference between the two equations?
  - Similarity: $y_t^{natural}$ is independent of monetary policy, and invariant to preference shocks $\{z_t\}$.

Equilibrium in the NKM

Introducing the output gap 5/5

- Subtracting the steady state (flexible price) mark-up from the average mark-up in period *t* (i.e. subtracting eq. (22) from eq.(21)) yields:

$$\mu_t - \mu = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t + \frac{1+\varphi}{1-\alpha}a_t + \log(1-\alpha) - \left[-\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)y_t^{natural} + \frac{1+\varphi}{1-\alpha}a_t + \log(1-\alpha)\right]$$

$$\Rightarrow \mu_t - \mu = \hat{\mu}_t = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\underbrace{\left(y_t - y_t^{natural}\right)}_{outputgap} = -\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\tilde{y}_t \qquad (24)$$

- Interpretation (of all parts of the expression):

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Equilibrium in the NKM

Finding the New Keynesian Phillips Curve 1/2

- Recall the expression for log deviations of average mark-up from ss (eq.(24)) and the inflation equation (18):

$$\mu_t - \mu = \hat{\mu}_t = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)\tilde{y}_t \qquad (24)$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \underbrace{\frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta}_{\lambda}\hat{\mu}_t \qquad (18)$$

- Combining these two yields the NKPC! Combining eq. (18) and eq. (24):

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \underbrace{\lambda\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right)}_{\kappa}\tilde{y}_t = \beta E_t\{\pi_{t+1}\} + \kappa\tilde{y}_t \qquad (25)$$

- The NKPC is **THE FIRST KEY EQUATION OF THE NKM!**
  - In order to find it, we have made use of all, but one of the eight equilibrium equations we brought with us from lecture 4.
  - How to interpret the NKPC?

Equilibrium in the NKM

Finding the New Keynesian Phillips Curve 2/2

- Recall the eight equations we brought with us from last week in order to describe and discuss equilibrium behavior in the NKM:

(1) $$\omega_t = \sigma c_t + \varphi n_t$$

(2) $$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

(3) $$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t,$$

(4) $$Y_t(i) = A_t N_t(i)^{1-\alpha},$$

(5) $$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{\hat{mc}_{t+k|t} + (p_{t+k} - p_{t-1}\}$$

(6) $$Y_t = C_t \Rightarrow y_t = c_t$$

(7) $$n_t = \frac{1}{(1-\alpha)}(y_t - a_t)$$

(8) $$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$$

(9) $$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

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  - Finding the Dynamic IS equation

- A New Keynesian treat☺

Equilibrium in the NKM

Finding the Dynamic IS equation 1/2

- Adding and subtracting current and expected natural level of output in equation (9) describes the consumption Euler equilibrium relationship in terms of the output gap:

$$y_t + y_t^{natural} - y_t^{natural} = E_t\left\{y_{t+1}^{natural}\right\} - E_t\left\{y_{t+1}^{natural}\right\} + E_t\{y_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

$$\Rightarrow \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} + E_t\left\{\Delta y_{t+1}^{natural}\right\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right) + \frac{1}{\sigma}(1-\rho_z)z \qquad (26)$$

- Next, we make use of the equilibrium real interest rate we found at lecture 3:

$$r_t^* = r_t^{natural} = \rho + (1-\rho_z)z_t + \sigma\psi_{ya}E_t\{\Delta a_{t+1}\} = \rho + (1-\rho_z)z_t + \sigma E_t\left\{\Delta y_{t+1}^{natural}\right\} \qquad (27)$$

and inserting for it in equation (25) yields the **dynamic IS-equation:**

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} + E_t\left\{\Delta y_{t+1}^{natural}\right\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \left[\underbrace{r_t^{natural} - (1-\rho_z)z_t - \sigma E_t\left\{\Delta y_{t+1}^{natural}\right\}}_{\rho}\right]\right) + \frac{1}{\sigma}(1-\rho_z)z_t$$

$$\Rightarrow \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^{natural}\right) \qquad (28)$$

Equilibrium in the NKM
Finding the Dynamic IS equation 2/2

- The DIS-eq is **THE SECOND KEY EQUATION OF THE NKM!**

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^{natural}\right) \quad (28)$$

Interpretation:

- From equation (23): $\Delta y_{t+1}^{natural} = \psi_{ya}^{natural} E_t\{\Delta a_{t+1}\}$, hence the equilibrium process of the natural real interest rate only varies with changes in preferences and technology:

$$r_t^{natural} = \rho + (1-\rho_z)z_t + \sigma E_t\{\Delta y_{t+1}^{natural}\} \quad (27)$$

Interpretation:

Summing up so far

- The following two equations are the non-policy building block of the NKM:

The NKPC

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \quad (25)$$

The DIS

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\Big(\underbrace{i_t - E_t\{\pi_{t+1}\}}_{r_t} - r_t^{natural}\Big) \quad (28)$$

$$r_t^{natural} = \rho + (1-\rho_z)z_t + \sigma E_t\{\Delta y_{t+1}^{natural}\} \quad (27)$$

The equilibrium process for the natural real interest rate

- Can say something about equilibrium behavior of, not only real, but also nominal variables!
  - The mechanism:

Summing up so far

- The classical Monetary Model
    - We found a unique solution for the equilibrium dynamics of real variables
    - Had to introduce monetary policy to say anything about nominal variables – no monetary policy behavior was more optimal than the next

- The New Keynesian model
- Need a description of how the nominal interest rate evolves over time in order to close the model
- Hence, the path for equilibrium behavior of real variables cannot be determined without such information!
- Solution: Must introduce monetary policy (lecture 7)
- Result: Non-neutrality of monetary policy **in the short run**
- The two models have equal properties in the long run

Plan for next three weeks

- Next week: Winter break

- In two weeks: Guest lecture by Kjetil Olsen

- Lectures 7-9: Back to the NKM, discussing monetary policy in the model
  - Lecture 7: Introducing an interest rate rule for monetary policy (Gali chapter 3.4.1)
  - Lecture 7: What are the effects of shocks under an interest rate rule? (Gali chapter 3.4.1)
  - Lecture 7: Learning a new method (2/2): The method of undetermined coefficients
  - Lectures 7 and 8: More on monetary policy in the basic model (Galí chapter 4, Clarida, Galí and Gertler (2000))
  - Lectures 8-9: Optimal policy in the NKM (Clarida, Galí and Gertler (1999))

Outline

- Finishing up from lecture 4:
  - Slides 32-43
- Equilibrium in the New Keynesian Model
  - Equilibrium equations to work with
  - Equilibrium behavior of inflation
  - Introducing the output gap
  - Finding the New Keynesian Phillips Curve
  - Finding the Dynamic IS equation
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Monetary Policy

(Advanced Monetary Economics)

ECON 4325